

**2004 HSC Notes from  
the Marking Centre  
Mathematics Extension 1**

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# 2004 HSC NOTES FROM THE MARKING CENTRE

## MATHEMATICS EXTENSION 1

### Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics Extension 1. It is based on comments provided by markers on each of the questions from the Mathematics Extension 1 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2004 Higher School Certificate Examination, the marking guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 1 course.

### Question 1

This question produced good attempts from nearly all candidates.

- (a) Many showed a lack of knowledge of the V-shape of this function. Those showing V-shapes often misplaced the vertex. The shading was often incorrect or non-existent. Many candidates did not show a number *plane* but attempted an  $x$  number *line* only.
- (b) Many simple algebraic errors were made, whether using the critical points method or the multiplication of both sides by  $(x + 1)^2$ . Very few candidates solved the inequality by considering the graphs of the hyperbola and the line  $y = 3$ . Those who did were generally totally successful. Generally those who considered the two cases, where the denominator is positive or negative, could not handle the method. Another common error was to square both sides of the inequality.
- (c) Many candidates could quote a correct formula, or equivalent, and obtain correct numerical expressions for both coordinates. Some correctly handled internal rather than external division.
- (d) This question was answered correctly by a high proportion of the candidature. Evaluating  $\sin^{-1}0.5$  as  $30$ ,  $30^\circ$ , or  $\frac{\pi}{3}$  were common errors. Some candidates did not recognise the expression as a standard integral.
- (e) Some candidates showed little familiarity with the method of integration by substitution. Eg  $\int x\sqrt{x-3}dx \rightarrow \int x\sqrt{u}dx$ ,  $u = x - 3 \rightarrow x = u - 3$  was seen often. Changing  $x\sqrt{x-3}$  to  $(u + 3)u$  was another common error. Not changing the limits or re-substituting for  $u$  led to inefficiency and errors. Many candidates who obtained the correct function to integrate (in  $u$ ) could not obtain the correct primitive because of errors involving the fractional powers.

Having made an error in changing the limits, candidates ended with expressions involving  $(-1)^{\frac{1}{2}}$  and evaluated by ignoring the negative sign.

## Question 2

- (a) Most candidates knew to use  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ , but had difficulty setting out their solution so that the denominator contained the term  $\frac{x}{5}$  and the constant  $\frac{1}{10}$  placed outside the limit. A common error was to put  $\sin \frac{x}{5} = \frac{1}{5} \sin x$  and to obtain an expression containing  $\frac{\sin 2x}{2x}$ . The most efficient and successfully used method was to use the fact that for small values of  $x$   $\sin \frac{x}{5} \approx \frac{x}{5}$ .
- (b) Most candidates either knew the derivative of  $\cos^{-1}x$  or applied the chain rule, but many were not able to put both together. Errors included forgetting to multiply by  $6x$ , omitting the negative sign, errors in squaring  $3x^2$ , trying to use the product rule and thinking that  $\cos^{-1}x = (\cos x)^{-1}$ .
- (c) This question was poorly done. Many candidates did not know the relationship between the tangent and the secant. The most common errors included use of Pythagoras' theorem or writing  $7(x + 7) = 12^2$ . The vast majority of the candidates who set up the correct equation  $x^2 + 7x - 144 = 0$  went on to find the correct solution  $x = 9$ , and rejected the incorrect solution  $x = -16$ .
- (d) (i) Most candidates were able to correctly find the value of  $A = 10$ . However, many found the value of  $\alpha$  in degrees rather than radians or found  $\tan^{-1} \frac{8}{6}$  rather than  $\tan^{-1} \frac{6}{8}$  (or equivalent). The value of  $\alpha$ , when put in radians, was given in various ways, including,  $\frac{37\pi}{180}$ ,  $\frac{\pi}{5}$ ,  $\frac{553\pi}{2700}$ ,  $0.205\pi$ .
- (ii) Most of those who used part (i) were able to obtain a solution for  $x$ , but many forgot about the second solution obtained from  $x - \alpha = \frac{5\pi}{3}$ . Again many candidates used degrees rather than radians or a mixture of both. A small number of candidates chose to find the solution using the 't method' or by using  $A \sin(x \pm \alpha)$ .
- (e) (i) This part was answered well. Common incorrect answers were  ${}^{16}P_4$ ,  ${}^9C_4 \times {}^7C_4$ .
- (ii) This part was answered well. However, a significant number of candidates used either  $\frac{{}^9P_4}{{}^{16}P_4}$  or evaluated  $\frac{9}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13}$ . Some candidates incorrectly stated the probability as  $\frac{{}^7C_4}{{}^{16}C_4}$  or tried to use binomial probability.

**Question 3**

- (a) Candidates who used the relationship  $\cos 8x = 2 \cos^2 4x + 1$  usually obtained a correct solution for the primitive function. Most candidates who attempted to use the relationship  $\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4} \sin 2x$  were unable to obtain a correct solution.
- (b) Candidates who used the remainder theorem were usually able to find the value of  $b$  in part (i) and then correctly solve an equation to determine the value of  $a$  in part (ii). Common errors involved attempts to evaluate  $P(1)$  and  $P(-3)$  when using the remainder theorem.

Weaker attempts used long division methods that were rarely successful. Many responses did not clearly identify the remainder as  $R(x) = 3x - 8$  and a frequent concluding statement was  $P(x) = Q(x) + 3x - 8$ . Other candidates believed that the remainder needed to be constant and solved expressions for  $a(x+1) + b = 0$ .

- (c) (i) Most candidates recognised the relationship  $x^2 + h^2 = 16$ . There were many incorrect attempts to express  $x$  in terms of  $h$ . Common errors included  $x = \sqrt{4 - h^2}$ , or  $x = 4 - h$ .
- (ii) Candidates who correctly stated a chain rule in terms of  $h$ ,  $x$  and  $t$  usually proceeded to a correct solution. Candidates who demonstrated knowledge of the rate of change method frequently made errors in differentiation or substitution. The implications of negative values of  $\frac{dx}{dh}$  and  $\frac{dh}{dt}$  were usually ignored. Some candidates successfully used implicit differentiation methods.
- (d) This question assessed skills in 3D trigonometry methods. Some responses showed little understanding of spatial concepts.
- (i) Correct responses usually involved recognition that triangle  $FAC$  is equilateral. Other candidates verified  $\angle FAC = 60^\circ$  using trigonometry.
- (ii) Candidates did not always offer sufficient explanation to show that  $FO = \sqrt{6}$  metres. Candidates often made unsupported statements or omitted intermediate steps in their solution. Attempts to use similar triangles often lacked adequate explanation.
- (iii) Candidates who attempted part (iii) usually recognised that  $\angle XFY = 2 \times \angle XFO$ . While many candidates calculated correctly that  $\angle XFO = \tan^{-1} \frac{1}{\sqrt{6}}$ , they were unable to calculate the size of  $\angle XFY$ . A common error was to state  $\angle XFY = \tan^{-1} \frac{2}{\sqrt{6}}$ .

**Question 4**

- (a) Many candidates submitted excellent responses including all the necessary components of an induction proof. Weaker responses were those that did not verify for  $n = 3$ , had an incorrect statement of the assumption for  $n = k$  or had an incorrect use of the assumption in the attempt

to prove the statement. Poor algebraic skills made it difficult or even impossible for some candidates to complete the proof. Rote learning of the formula  $S_k + T_{k+1} = S_{k+1}$  led many candidates to treat the expression as a sum rather than a product, thus making completion of the proof impossible.

- (b) (i) This part was answered very well by the majority of candidates, but many wasted time deriving the equation despite being told not to do so. Most were able to correctly substitute for  $R$ , or to solve the equations simultaneously. Candidates are reminded that when asked to show a result they are expected to actually demonstrate every step. Many correctly found the  $x$  coordinate and then simply stated  $y = apq$  with no justification. Others simply ignored the fact that their expression did not yield the required result eg  $x(p-q) = a(q^2-p^2)$ .
- (ii) This part was challenging for most candidates, and many did not attempt it. Some candidates were able to establish that  $pq = -4$  but then were unable to continue or they spent much time and effort trying to find a relationship between  $x$  and  $y$ . The most common error otherwise was to assume that  $pq = -1$  and then state that the locus must be the directrix.
- (c) (i) Most candidates correctly evaluated  $1 - \left(\frac{9}{10}\right)^7$ .
- (ii) Most candidates stated the binomial probabilities correctly and then proceeded to evaluate and compare them. For candidates who did not evaluate the probabilities the better responses were those that considered a ratio that was then simplified directly and compared to 1. Some of the poorer responses used the wrong indices, usually 2 and 8, or the wrong probability.
- (iii) Many candidates produced excellent responses. In some cases the algebra involved required much care and accuracy. In general, the candidates who simplified their original probability expressions immediately were more likely to end up with the correct answer. Some candidates used  $\frac{n-k+1}{k} \times \frac{b}{a} > 1$  with success but many reversed  $a$  and  $b$  for an incorrect result.

### Question 5

Candidates are reminded to show all working and to take care with notation.

- (a) (i) This part was generally well done by candidates who understood that acceleration was equal to  $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ . Candidates were required to correctly evaluate the constant of integration and to clearly show the factorisation of  $v^2 = x^4 + 2x^2 + 1$ .
- (ii) Candidates are reminded to use the table of standard integrals and take care when evaluating the constants of integration. Better responses demonstrated the use of radians rather than degrees.
- (b) (i) Better responses used the reflection of  $y = f(x)$  in the line  $y = x$ . Care needs to be taken when drawing graphs to ensure the significant features are clearly indicated.



- (ii) Many candidates were able to give the correct domain, either from their correct graph in (i) or by understanding the link to the range of  $f(x)$ .
- (iii) Most candidates correctly interchanged  $x$  and  $y$  before attempting to make  $y$  the subject. Candidates who used function notation often had problems correctly interchanging the variables.
- (iv) Better responses to this part demonstrated an understanding that  $y = f(x)$  and  $y = f^{-1}(x)$  intersect on the line  $y = x$ . Candidates who attempted to solve  $y = f(x)$  and  $y = f^{-1}(x)$  simultaneously were rarely successful. Candidates are reminded to use the number of marks allocated as an indication of the length of the solution.
- (v) Some candidates did not see the link to part (iv) of the question. Better responses clearly showed the correct substitution of  $x=0.5$  into the function and its derivative, the correct evaluations, and the subsequent use of Newton's method.

### Question 6

Overall, responses to this question were quite good.

- (a) Candidates need to be reminded about the value of drawing the diagram and actually using it to assist in their response. Use of symbols such as  $\angle 1$  or  $\angle 2$  etc are meaningless unless accompanied by a diagram.
  - (i) Many candidates did not attempt this part. Some were not able to explain their reasoning. Many candidates named angles incorrectly and many started off by assuming that  $AD$  or  $BD$  were diameters, or that  $E$  was the centre of the circle. Some also took  $ABF$  as a tangent and then applied inappropriate theorems.
  - (ii) There were many non-attempts for this part, even by those candidates who completed (i). Others who could not find the size of  $\angle DBF$  still stated the length of  $AD$  correctly.
- (b) The candidates who attempted this part, on the whole, did quite well. Usually it was the algebra or arithmetic that led to incorrect answers. Some candidates wasted a lot of time deriving all the motion equations. Candidates need to be reminded that in 'show' questions they must give clear reasons for their statements.
  - (i) Most candidates were able to do this quite easily, but not always in the most efficient way.
  - (ii) Some candidates did not see the connection between this part and (i) and consequently took a page to answer the question, sometimes making arithmetic errors.
  - (iii) Most candidates did this part quite well but were not able to show their use of  $v^2=80g$ .
  - (iv) Some candidates did not appear to know that  $\sec^2\theta = \tan^2\theta + 1$  or they made arithmetic mistakes. Some approached this part by solving the equation but did not show that both their solutions satisfied the given conditions.

- (v) Most candidates merely solved the equation given in (iv) without too much thought. Others did not realise that solving this would give part of the solution and went on to solve something else, usually an incorrect equation. Very few could interpret the problem and realise that there would be two sets of conditions when the water hits the front of the wall.

### Question 7

There were a number of easy marks to be gained in both parts of this question.

- (a) (i) Many candidates successfully drew a graph. Common errors included confusion between the period and  $n$  and confusion between  $t = 0$  and 2 am. It is recommended that candidates use correct terminology for the centre, amplitude and period, rather than personal notation.
- (ii) Generally well done by candidates who attempted this part. However, a significant number of candidates substituted  $t = 2\frac{1}{12}$  to show that  $y = 8.5$ , rather than solving the equation for  $y = 8.5$ . In this way candidates showed that the depth was 8.5 m at this time, but not that it was the earliest time. The most common errors included working in degrees and difficulties with conversions from 2.08 $\dot{3}$  to hours and minutes. Many candidates who worked in degrees then manipulated their answer to obtain the time 4.05 am.
- (iii) Candidates were often unable to correctly account for the time differences. Many attempted to account for the one hour difference in a variety of ways – by adding/subtracting their time to/from 2 am or 1 am, by adjusting  $t$  in their equation, by adjusting the period, while frequently ignoring the 20 minute time difference. Some candidates solved the equation  $6 = 7 + 3\cos\left(\frac{4\pi t}{25}\right)$ , but algebraic errors such as  $\cos\left(\frac{4\pi t}{25}\right) = \frac{1}{3}$  were common. Another common error was to work in degrees. Some candidates attempted to account for the time differences by adjusting the original equation to  $6 = 7 + 3\cos\left(\frac{4\pi t}{25} \pm 1\right)$ .
- (b) (i) Candidates who recognised the geometric series were generally successful in gaining the mark, while those who substituted  $a = (1+x)^{n-1}$  and  $r = (1+x)^{-1}$  mostly found the algebraic manipulation too demanding. A number of candidates who chose to establish the result by mathematical induction were successful, but candidates are encouraged to consider the mark allocation to determine the amount of working required in a solution. A few candidates factorised  $(1+x)^n - 1$ , but there were some elegant solutions expanding the LHS as  $[(1+x)-1][(1+x)^{n-1} + \dots + 1]$ . Most candidates who attempted this part gained the mark.
- (ii) Candidates were required to indicate that they were equating the coefficients of  $x^k$  in the identity from (i). Generally, candidates did not clearly state that they were equating

the coefficients of  $x^k$  but referred to the  $k$  th term, the  $k$  term etc. A number of candidates tried unsuccessfully to prove the result by expanding the binomial coefficients on the LHS.

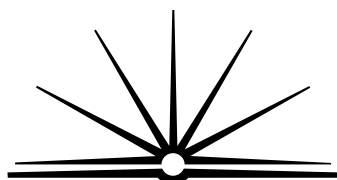
- (iii) Many candidates were successful in showing this result. However, students should be encouraged not to work on both sides of the identity, but to simplify the LHS and RHS separately.
- (iv) Very few candidates were able to establish this result, although some did correctly differentiate both sides of the identity in (i). Common errors in the differentiation included misuse of the product rule or differentiation of the RHS to give  $n(1+x)^{n-1} - 1$ . Where the derivative was correct, many candidates were not able to identify which coefficients to compare and did not recognise the links to parts (ii) and (iii).

# Mathematics Extension 1

## 2004 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	1.2, 4.4	P5
1 (b)	3	1.4 E	PE3
1 (c)	2	6.7E	P4
1 (d)	2	15.5	HE4
1 (e)	3	11.5	HE6
2 (a)	2	13.4	H5
2 (b)	2	15.5	HE4
2 (c)	2	2.9, 2.10	PE3, PE2
2 (d) (i)	2	5.9, 13.1	HE3, H5
2 (d) (ii)	2	5.9, 13.1	HE3, H5
2 (e) (i)	1	18.1	PE3
2 (e) (ii)	1	18.1, 3.1	PE3, H5
3 (a)	2	13.6 E	H5, HE6
3 (b) (i)	1	16.2	PE3
3 (b) (ii)	2	16.2	PE3
3 (c) (i)	1	2.3	H4, P4
3 (c) (ii)	3	14.1	HE5
3 (d) (i)	1	5.3, 5.6, 2.3	PE2, P4
3 (d) (ii)	1	5.3, 5.6	PE2, P4
3 (d) (iii)	1	5.6	PE2, P4, H5
4 (a)	3	7.4	H2, HE2
4 (b) (i)	2	9.6	PE3
4 (b) (ii)	2	9.6	PE3
4 (c) (i)	1	18.2	H5, HE3

Question	Marks	Content	Syllabus outcomes
4 (c) (ii)	2	18.2	HE3
4 (c) (iii)	2	18.2	HE3
5 (a) (i)	2	14.3E	HE5
5 (a) (ii)	3	14.3E	HE5
5 (b) (i)	1	15.1	HE4
5 (b) (ii)	1	15.1	HE4
5 (b) (iii)	2	15.1	HE4
5 (b) (iv)	1	15.1	HE4
5 (b) (v)	2	16.4	HE4, H5
6 (a) (i)	2	2.10	PE3, H5
6 (a) (ii)	1	2.10	PE3, H5
6 (b) (i)	2	14.3E	HE3
6 (b) (ii)	1	14.3E	HE3
6 (b) (iii)	2	14.3E	HE3
6 (b) (iv)	2	14.3E	HE3
6 (b) (v)	2	14.3E, 9.4, 9.1	HE3, H5
7 (a) (i)	2	14.4.E	HE3
7 (a) (ii)	2	14.4E	HE3
7 (a) (iii)	2	14.4.E	HE3
7 (b) (i)	1	17.3	HE3
7 (b) (ii)	1	17.3	HE3
7 (b) (iii)	1	17.3	HE3
7 (b) (iv)	3	17.3	HE3



**B O A R D O F S T U D I E S**  
NEW SOUTH WALES

## 2004 HSC Mathematics Extension 1 Marking Guidelines

### Question 1 (a)

*Outcomes assessed: P5*

#### MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• An answer which displays some understanding of the graph of the absolute value of a function or equivalent merit	1

### Question 1 (b)

*Outcomes assessed: PE3*

#### MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Identifies $x = \frac{1}{3}$ and $x = -1$ as values of significance for this inequality or equivalent progress	2
• Identifies one of the two intervals in the correct solution or equivalent merit	1

**Question 1 (c)***Outcomes assessed: P4***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• One coordinate correctly calculated or finds the point which divides $AB$ internally in the ratio 5:2 or equivalent merit	1

**Question 1 (d)***Outcomes assessed: HE4***MARKING GUIDELINES**

Criteria	Marks
• Correct solution (numerical equivalent is acceptable)	2
• Correct primitive or equivalent merit	1

**Question 1 (e)***Outcomes assessed: HE6***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	3
• Substitution is carried out correctly or equivalent merit	2
• Shows some understanding of substitution (e.g. replacing $x - 3$ by $u$ and $dx$ by $du$ or changing limits of integration)	1

**Question 2 (a)***Outcomes assessed: H5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Attempts to apply the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ in this context	1

**Question 2 (b)***Outcomes assessed: HE4***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Displays understanding of chain rule or $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	1

**Question 2 (c)***Outcomes assessed: PE3, PE2***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Applies the theorem about the length of tangents and secants to this situation	1

**Question 2 (d) (i)***Outcomes assessed: HE3, H5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution. If no exact expression for $\alpha$ is given, the numerical approximation must be in circular measure	2
• Finds the value of $A$ or $\alpha$ , or equivalent merit	1

**Question 2 (d) (ii)***Outcomes assessed: HE3, H5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution using values for $A$ and $\alpha$ found in part (i)	2
• Obtains one of the two values for $x$ , or equivalent merit	1



**Question 2 (e) (i)***Outcomes assessed: PE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct numerical expression	1

**Question 2 (e) (ii)***Outcomes assessed: H5, PE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct numerical expression using answer found in part (i)	1

**Question 3 (a)***Outcomes assessed: H5, HE6***MARKING GUIDELINES**

Criteria	Marks
• Correct primitive	2
• Correctly rewrites $\cos^2 4x$ in terms of $\cos 8x$ or correctly finds the primitive of an expression of similar complexity found as a result of an attempt to apply the double angle formula	1

**Question 3 (b) (i)***Outcomes assessed: PE3***MARKING GUIDELINES**

Criteria	Marks
• Correct answer	1

**Question 3 (b) (ii)***Outcomes assessed: PE3***MARKING GUIDELINES**

Criteria	Marks
• Correct answer	2
• Finds $a$ or recognises that the remainder will be $a(x + 1) + b$	1

**Question 3 (c) (i)***Outcomes assessed: H4, P4***MARKING GUIDELINES**

Criteria	Marks
• Correct answer	1

**Question 3 (c) (ii)***Outcomes assessed: HE5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	3
• Finds an expression for $\frac{dx}{dt}$ and interprets the given information to find $\frac{dh}{dt}$ when $h = 1$ OR equivalent	2
• Finds an expression for $\frac{dx}{dt}$ OR writes down the value of $\frac{dh}{dt}$ when $h = 1$ OR equivalent	1

**Question 3 (d) (i)***Outcomes assessed: PE2, P4***MARKING GUIDELINES**

Criteria	Marks
• Gives a correct explanation	1

**Question 3 (d) (ii)***Outcomes assessed: PE2, P4***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	1

**Question 3 (d) (iii)***Outcomes assessed: PE2, P4, H5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	1

**Question 4 (a)***Outcomes assessed: H2, HE2***MARKING GUIDELINES**

Criteria	Marks
• Verifies the case when $n = 3$ and establishes that if the formula holds for an integer $n$ it also holds for $n + 1$	3
• Verifies one case and attempts to prove the inductive step, or establishes that if the formula holds for an integer $n$ it also holds for $n + 1$ , or equivalent merit	2
• Verifies one case or attempts to prove the inductive step	1

**Question 4 (b) (i)***Outcomes assessed: PE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Establishes that $R$ lies on one of the tangents, or finds the equation of both tangents and attempts to solve simultaneously, or equivalent merit	1

**Question 4 (b) (ii)***Outcomes assessed: PE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Deduces the relationship between $p$ and $q$ , or equivalent merit	1

**Question 4 (c) (i)***Outcomes assessed: H5, HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct answer	1

**Question 4 (c) (ii)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Obtains a correct expression for one of the probabilities	1

**Question 4 (c) (iii)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Shows that, for the probability to be greater, Katie must participate for more than 29 weeks	2
• Deduces that $\binom{n}{3} \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{n-3} > \binom{n}{2} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{n-2}$ or equivalent progress	1

**Question 5 (a) (i)***Outcomes assessed: HE5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Obtains $\frac{1}{2}v^2 = \frac{x^4}{2} + x^2(+C)$ , or equivalent progress	1

**Question 5 (a) (ii)***Outcomes assessed: HE5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	3
• Deduces that $t = \tan^{-1} x + C$ and evaluates $C$ or equivalent	2
• Deduces that $t = \tan^{-1} x (+C)$ or equivalent	1

**Question 5 (b) (i)***Outcomes assessed: HE4***MARKING GUIDELINES**

Criteria	Marks
• Sketch which is recognisably a reflection of the original curve in the line $y = x$	1

**Question 5 (b) (ii)***Outcomes assessed: HE4***MARKING GUIDELINES**

Criteria	Marks
• Correct answer	1

**Question 5 (b) (iii)***Outcomes assessed: HE4***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct answer	2
• Indicates that $x = \frac{1}{1+y^2}$ or equivalent merit	1

**Question 5 (b) (iv)***Outcomes assessed: HE4***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct explanation	1

**Question 5 (b) (v)***Outcomes assessed: HE4, H5***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	2
• Solution demonstrates some knowledge of Newton's method	1

**Question 6 (a) (i)***Outcomes assessed: PE3, H5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Observes that $\angle ABD = \angle ACD$ or equivalent progress	1

**Question 6 (a) (ii)***Outcomes assessed: PE3, H5***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	1

**Question 6 (b) (i)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Computes $t$ when $y = 0$ or equivalent progress	1

**Question 6 (b) (ii)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	1



**Question 6 (b) (iii)***Outcomes assessed: HE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	2
• Makes significant progress towards eliminating $t$ or $v$	1

**Question 6 (b) (iv)***Outcomes assessed: HE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	2
• Substitutes $y = 20$ and $x = 40$ in the equation in (iii) or equivalent	1

**Question 6 (b) (v)***Outcomes assessed: HE3, H5***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	2
• Deduces one of the critical angles other than $15^\circ$ or equivalent merit	1

**Question 7 (a) (i)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Solution makes reference to the correct period, amplitude or centre of motion or equivalent	1

**Question 7 (a) (ii)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Deduces that $\cos\left(\frac{4\pi t}{25}\right) \leq \frac{1}{2}$ or equivalent	1

**Question 7 (a) (iii)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	2
• Accounts for the time differences or computes that the tidal depth at the wharf is 6m at 5:48 or equivalent progress	1

**Question 7 (b) (i)***Outcomes assessed: HE3***MARKING GUIDELINES**

Criteria	Marks
• Correct solution	1

**Question 7 (b) (ii)***Outcomes assessed: HE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	1

**Question 7 (b) (iii)***Outcomes assessed: HE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	1

**Question 7 (b) (iv)***Outcomes assessed: HE3***MARKING GUIDELINES**

<b>Criteria</b>	<b>Marks</b>
• Correct solution	3
• Correctly differentiates both sides and makes significant progress in identifying the coefficient of $x^k$ on both sides or equivalent merit	2
• Correctly differentiates both sides	1