2009 HSC Notes from the Marking Centre Mathematics Extension 1

Published by Board of Studies NSW
GPO Box 5300
Sydney 2001
Australia

Tel: (02) 9367 8111
Fax: (02) 9367 8484
Internet: www.boardofstudies.nsw.edu.au

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2009 HSC NOTES FROM THE MARKING CENTRE
MATHEMATICS EXTENSION 1

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics Extension 1. It contains comments on candidate responses to the 2009 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2009 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 1.

Teachers and students are advised that, in December 2008, the Board of Studies approved changes to the examination specifications and assessment requirements for a number of courses. These changes will be implemented for the 2010 HSC cohort. Information on a course-by-course basis is available on the Board's website at www.boardofstudies.nsw.edu.au/syllabus_hsc

Question 1

(a) Most candidates obtained the $(2x+3)$ factor but many then failed to complete the factorisation or had to use polynomial division to find the other factor.

(b) This part was generally well done.

(c) Most candidates obtained 2, although a significant number achieved this through incorrectly rewriting $\sin 2x$ as $2\sin x$. Those who used l'Hopital's rule were usually successful but the method of using the small angle theorem, that is $\sin 2x \approx 2x$, mostly did not contain sufficient explanation.

(d) In weaker responses, candidates failed to recognise the need to consider the possibility that the x term from the denominator could be negative, or they treated this part as an absolute value inequality. There were a number of methods used, with the critical point method being the most successful; however, candidates using this method needed to make sure they solved the equality, not the inequality, for the second critical point. Other methods commonly used included: multiplying both sides by a square (but a significant number of candidates could either not complete the algebra leading to a correct quadratic inequality or could not solve a quadratic inequality); considering the two cases $x > 0$ and $x < 0$, with some having difficulties drawing the correct conclusion from the resulting inequalities; or writing the inequality as $\frac{x+3}{2x} - 1 > 0$ leading to $2x(x-3) > 0$, which also resulted in poor algebraic manipulation and on a number of occasions was poorly explained. A few candidates used a graphical method successfully.

(e) Most candidates successfully applied the product rule but the differentiation of $\cos^2 x$ was poorly done. A significant number of candidates tried to replace $\cos^2 x$ using a double-angle formula, but most then made mistakes in the differentiation process.

(f) A significant number of responses performed the substitution and reached $k\int_1^9 e^u du$ but could not then complete the integration, with $\left[\frac{e^{u+1}}{u+1}\right]$ and $\left[\frac{e^u}{u}\right]$ being common incorrect responses. The limits were not always changed correctly and sometimes they were not changed at all.

Question 2

(a) The remainder theorem was well understood and used in most responses. Very few substituted incorrect values for x. A small number used the division process, but were often unsuccessful in obtaining the correct solution as they were unable to equate the correct remainder with the given values. Although many candidates had the correct equations to solve, a high percentage of them did not successfully complete the solution. Some made simple arithmetic errors and mistakes with algebra, especially when transferring terms to the other side. Only a small number of candidates attempted to use the factor theorem.

(b) Those candidates who understood how to use the auxiliary angle method performed well with few subsequent errors. It was clear when that method was not known, with the candidates not even attempting it or being able to even write $A\sin(x+\alpha)=5$, even after finding the values for A and α. Many candidates were unable to work in radians, as was required. A surprising number of candidates were unable to find x from the equation $x+\alpha=\frac{\pi}{2}$ or a similar expression, often with a mixture of degrees and radians.

(c) The lead-in, part (i), where the candidates were given the result, was instrumental in allowing a significant number of candidates to correctly obtain the equation of the tangent in part (ii). The progression through the three parts saw most obtaining the correct coordinates of the point R. Yet again many arithmetic errors were seen. Most candidates knew how to find the locus of R.

Question 3

(a) (i) Most candidates recognised that $e^x > 0$ but failed to recognise that this implied that $f(x) > \frac{3}{4}$. Another common error was to write $f(x) \geq \frac{3}{4}$.

(ii) The most successful method to find the inverse function was to interchange x and y and then make y the subject.

(b) (i) In better responses, candidates provided neat and accurate graphs with the points of intersection clearly marked. In the weaker responses, candidates demonstrated that they knew the shape of a cosine graph but had difficulty with the amplitude and/or period.

(ii) Nearly all candidates established the relationship between the points of intersection from the graph in part (i) and the number of solutions to $2\cos 2x = x + 1$.

(iii) In the better responses, candidates defined a suitable function $f(x)$, calculated $f'(x)$ and then evaluated both $f(0.4)$ and $f'(0.4)$ before they used these values in Newton's formula. In weaker responses, candidates differentiated $f(x)$ incorrectly or made careless errors with negative signs when substituting $f'(0.4)$ into Newton's formula. When evaluating trigonometric functions it is important to remember to have the calculator in radians mode.

(c) (i) In the more successful responses, candidates manipulated the right-hand side by successfully substituting one of the three forms of $\cos 2\theta$ from the double-angle formulae in the numerator and denominator to arrive at the left-hand side. Candidates are advised that in questions that require proof there must be clear progress from one line to the next.

(ii) Better responses successfully used the link between parts (i) and (ii).

Question 4

(a) (i) Some candidates did not realise that this was a binomial probability question, and used only fractions and powers. Some confused the difference between 'correct' and 'incorrect'; some used fifths rather than quarters. Many wrote the correct expression, but did not evaluate the binomial coefficients.

(ii) This part was generally well done, but some responses omitted one term when adding or subtracting terms. Some multiplied the terms instead of adding.

(iii) Some candidates tried adding the five terms, but this approach made a 1-mark part very laborious and time-consuming, and such approaches often lost the mark by leaving out one of the terms. The most common error was to use $1-\left(\frac{3}{4}\right)^5$ rather than $1-\left(\frac{1}{4}\right)^5$.

(b) (i) Candidates were required to show that the function was even by demonstrating that $f(-x) = f(x)$, but a number of candidates simply substituted various random numbers, or just stated the definition of an even function without any further work. Candidates are reminded that the substitution of a negative term is best done by using parentheses; the claim that $-x^4 = x^4$ was very common.

(ii) Even though most candidates could perform the limit calculation, they failed to state that the equation of the asymptote was $y = 1$. Many candidates left their answer as a limit: as y approaches 1 or zero. A large number of candidates answered $y = 0$ or $x = 1$. Very few candidates bothered to state the equation on their graph.

(iii) A considerable number of candidates failed to differentiate properly, making a sensible final answer very difficult to reach. Common errors included reversing the terms on the numerator, leaving parentheses out of the numerator or neglecting to include the denominator at all.

Some attempted to use the product rule with negative indices rather than the quotient rule, but this approach was generally very poorly done.

Most candidates looked for the values where the derivative was equal to zero to find stationary points, but many did not realise that the denominator could be ignored at this stage, and decided to try simplifying the whole fraction, usually making serious errors. Some had difficulty factorising $6x^5 - 12x^3 - 18x = 0$ to $6x(x^2 - 3)(x^2 + 1) = 0$, others could not find the roots, while others found five roots, usually including ± 1.

A considerable number of candidates failed to consider the two roots at $\pm\sqrt{3}$, and only considered the positive answer. This ignored the fact that the function was even, as stated in part (i). Failure to take out a factor of x meant that some did not find the $(0,0)$ stationary point.

(iv) Most candidates placed three stationary points and an asymptote on the graph, but did not reconcile how to draw the graph passing through, as well as approaching, the asymptote. Many candidates seemed to ignore the fact that they had been asked to show that the function was even in part (i), or did not know that the graph of an even function is symmetrical about the y-axis. Some showed vertical asymptotes, and a large number included a cusp at $(0,0)$, especially if they had not identified it as a stationary point in part (iii) above.

Candidates are reminded that placing a scale on their axes, and writing the equation of the asymptote on the graph provides information that could earn marks.

Question 5

(a) (i) Many candidates correctly attempted a solution by integrating $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -n^2x$. As the initial conditions were inferred in the question rather than stated explicitly, candidates then often had difficulty evaluating the constant of integration, and many made a second attempt at their solution. There were a variety of alternative solutions, for example differentiating $\frac{1}{2}v^2$ to show that $\ddot{x} = -n^2x$ or differentiating $x = \cos(nt + a)$ to show that $v^2 = n^2(a^2 - x^2)$. These alternative solutions rarely gained full marks unless executed carefully by accounting for the initial conditions or showing that their equation for displacement demonstrated that the particle was in simple harmonic motion.

Mid-range responses did not find the constant of integration. In the better responses, candidates were often efficient in their solutions by using definite integrals from the equation $v\frac{dv}{dx} = -n^2x$.

(ii) A number of candidates stated where the maximum speed occurred but not what that maximum speed was. Candidates who approached their solution by substituting into $v^2 = n^2(a^2 - x^2)$ often made an incorrect substitution, made algebraic errors, or found that $v^2 = a^2n^2$ without stating the maximum speed. A number of candidates simply stated that the maximum speed was an, possibly indicating an understanding of the amplitude of the equation for velocity as a function of time.

(iii) A number of candidates stated where the maximum acceleration occurred or gave the maximum acceleration as $-n^2x$. A number of candidates were able to simply state that the maximum acceleration was an^2, possibly indicating an understanding of the amplitude of the equation for acceleration as a function of time.

(iv) Many candidates misread the directions in the question to write down a formula and instead derived the required formula. Many of these candidates successfully integrated $\frac{dx}{dt}$, but their lack of understanding of terminology meant that they produced a one- or two-page solution instead of a simple statement that $x = a\sin nt$, and this wasted a lot of time. Many candidates then made no attempt at the second part of the question, perhaps not realising that they had not fully answered the question. Candidates must be careful in multiple-answer parts to ensure that they do not omit a section through carelessness. Many candidates who attempted the second part of the question were successful in gaining this mark. Candidates who gave the general solution then often had difficulty interpreting the request for the first time that the particle's speed is half its maximum speed. Some weaker responses indicated that the speed will be half of the maximum speed at $x = \frac{a}{2}$, that is, at half the distance between the centre of motion and the extremity.

(b) (i) Candidates who succeeded in this part used a number of different approaches. Those who drew a diagram and provided detail on the diagram were generally more successful. The most efficient approach was often to use trigonometry in a right-angle triangle to find half the length of the top of the triangular face using $\tan 60° = \frac{x}{h}$, and then to find the volume as $\frac{1}{2} \times 2x \times h \times 10 = 10\sqrt{3}\, h^2$. Many candidates displayed poor spatial understanding, being confused about finding the volume at depth h and instead used the depth 3 metres in their calculations. Candidates also commonly considered h to be the slant height of the cross-section instead of the perpendicular height. Another common error was to find the volume of a pyramid using $V = \frac{1}{3}Ah$. Some solutions were marred by the use of incorrect trigonometric ratios. There was obvious confusion in some responses when attempting to use sine or cosine rule, about which triangle(s) they were working with. For example, a number of candidates stated that $\frac{\sin 120°}{x} = \frac{\sin 30°}{h}$ where x was the length of top of the triangular face at depth h.

(ii) Candidates who were successful in part (i) usually showed the result in part (ii). Many candidates gained the mark easily if they indicated the correct length and width of the surface area without any contradictory working.

(iii) Many candidates gained one mark for this part if they stated an appropriate chain rule, even if they failed to apply it correctly or to simplify it completely. To gain both marks candidates were required to substitute for A or to evaluate their chain rule in terms of A in order to find the correct rate of change $\frac{dh}{dt} = -k$. Many candidates gave their result as $\frac{dh}{dt} = \frac{-kA}{20\sqrt{3}\,h}$ and so did not realise $\frac{dh}{dt}$ was a constant, making part (iv) difficult to answer successfully. Candidates should use correct notation; some introduced their own variable and unless it was clear that their variable referred to the height *h*, they could not gain the mark for a correct chain rule.

(iv) Few candidates recognised or interpreted the constant rate of change $\frac{dh}{dt} = -k$. Candidates who attempted this part often integrated their rate of change, some successfully finding that when the height was 1 metre the elapsed time was 200 days but then neglecting to find the time taken for the height to decrease from 2 metres to 1 metre. Candidates are again reminded to look at the mark value of a part when approaching their solution. In some weaker responses, candidates also attempted to solve an exponential decay equation, $V = Ae^{-kt}$, showing little understanding of the problem. Some candidates who were incorrect in part (i) then did not find the rate of change to be constant, valiantly integrated their rate of change, some successfully.

Question 6

(a) (i) Most candidates who equated the expressions for x_1 and x_2 derived the correct expression for T. Since this part asked the candidates to show a result, it was difficult to award full marks to candidates who provided insufficient working. As a result, few candidates gained full marks if they had not made R the subject of the equation at some stage.

(ii) In better responses, candidates showed $y_1 - y_2 = 0$ at time T, but few candidates used this method. Many candidates substituted T into y_1 and y_2 in an attempt to show that the two expressions were equivalent, but only a few of them were successful. Those who were not successful either did not make the substitution $\tan\theta = \frac{h}{R}$, or made algebraic mistakes. Most of the remaining candidates equated the expressions for y_1 and y_2. Of those, many found the correct expression $\frac{h}{(U+V)\sin\theta}$, but few correctly showed that $\frac{h}{(U+V)\sin\theta} = T$. A common mistake was to equate y_1 and y_2, and substitute the value of T into the resulting expression.

(iii) Candidates are reminded that in a question requiring them to show a result, sufficient steps must be given so that the examiner can be certain that the candidate has actually derived the result and not merely written down the expression given in the examination paper.

(b) (i) Better responses gave the correct number of terms in the series, that is, $n - r + 1$. Candidates are reminded that if they are required to show a particular result then all steps of working must be shown and all statements justified. Hence, candidates needed to give an explanation as to why the coefficient of x^r was $\binom{n+1}{r+1}$ so that the examiners could see that the candidate was not merely writing down the result given in the examination question. Candidates are reminded not to make deliberate incorrect changes to their working so as to arrive at the result given in the examination question.

A common mistake was that candidates assumed the number of terms was n. This led to the (incorrect) expression, $\frac{(1+x)^{n+r} - (1+x)^r}{x}$. Candidates who then tried to convince the examiners that the coefficient of x^r was $\binom{n+1}{r+1}$ could not gain the mark allocated for this part of the question. However, candidates who gave the correct value for the coefficient of x^r according to their working and an acceptable explanation could be awarded the mark.

(ii) Only the better responses gave adequate explanations. In part (1), examiners were not able to tell if the candidates who only wrote 'from *n* choose 2' were describing why the number of intervals was $\binom{n}{2}$ or if they were just stating the meaning of the symbol $\binom{n}{2}$. Better responses indicated that there were n points on the line $y = x$ from which 2 were chosen. It was difficult to award marks to responses that did not use the word 'choose' in their explanation.

In part (2), candidates needed to word their solutions carefully to convince the examiners that the total number of intervals is found by adding the number of intervals along each diagonal. Many candidates may have realised that this is the case, but if they did no more than describe each term they could not be awarded the mark. It was difficult to award the mark to candidates who did not indicate that $\binom{2}{2}$ represented the number of intervals on the lines containing 2 points, and that the largest number of intervals occurred on the line $y = x$.

(iii) Only the best responses correctly applied the result given in (i). In some instances, candidates who made an incorrect application of the result in (i) and then continued with correct working out received a mark. Candidates are reminded that marks will not be awarded when deliberate incorrect changes to working out are made so as to arrive at the result given in the examination question. Hence those who made an incorrect application of the result in (i) and then obtained the expression given in the examination paper could not receive the mark.

Question 7

The great majority of candidates attempted and gained marks in this question. Candidates are reminded to read the question carefully, show all necessary working, take care with notation and use the numbers of marks allocated to each part of the question as a guide to the working or reasoning required.

(a) (i) Many candidates who attempted this part showed some understanding of differentiation from first principles, although the use of a limit was often not clear and prevented some candidates from gaining the mark.

(ii) Many candidates did not realise that the proof of the initial case had already been done in part (i) and therefore wasted time proving the result again. The induction process was generally understood by most, enabling most candidates to gain at least one mark. Although this part instructed candidates to use the product rule, many could not as they did not realise that $\frac{d}{dx}\left(x^{k+1}\right)=\frac{d}{dx}\left(x \times x^{k}\right)$.

(b) (i) Responses that included a diagram and labelled the angles were generally successful in this part. Some candidates had difficulty expressing θ as a difference between two angles. Common errors included incorrect tangent ratios, poor notation and algebraic mistakes.

(ii) This part required candidates to use the quotient rule and chain rule, and to differentiate an inverse tangent function. Better responses clearly indicated the use of the following: $\frac{d\theta}{dx}=\frac{1}{1+\left[f(x)\right]^2}\times f'(x)$. Candidates who realised that the solutions are found by solving $f'(x)=0$ are reminded to show sufficient supporting evidence to gain full marks. A few candidates attempted implicit differentiation, usually with limited success.

(c) (i) The lack of geometric reasoning to support written observations prevented many candidates from gaining full marks. Very few candidates recognised that ϕ was the exterior angle of triangle PSR and therefore had difficulty explaining the relationship between θ and ϕ.

(ii) This part of the question specifically required candidates to use circle geometry reasoning to find the distance of P from the building. A diagram with the point of intersection of the base of the wall with the ground clearly labelled assisted some candidates to see that $TO^2 = OR \times OQ$ and hence produce the correct result. Candidates who used their result from part (b)(ii) without providing reasoning involving the tangent at T could not gain the mark.