2009 HSC Notes from the Marking Centre Mathematics

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MATHEMATICS

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics. It contains comments on candidate responses to the 2009 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2009 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of Mathematics.

Candidates are advised to read the questions carefully and set out their working clearly. In answering parts of questions, candidates should state the relevant formulas and the information they use to substitute into the formulas. In general, candidates who show working make fewer mistakes. When mistakes are made, marks can be awarded for the working shown. If a question part is worth more than one mark, working is expected to be shown. Any rough working should be included in the answer booklet for the question to which it applies.

Teachers and students are advised that, in December 2008, the Board of Studies approved changes to the examination specifications and assessment requirements for a number of courses. These changes will be implemented for the 2010 HSC cohort. Information on a course-by-course basis is available on the Board's website at www.boardofstudies.nsw.edu.au/syllabus_hsc

Question 1

(a) Although most candidates sketched a line, many responses failed to correctly display the intercepts on both axes. Candidates are reminded that a response can only gain full marks if it correctly addresses all aspects of the question.

(b) Most candidates successfully adopted the simple approach of multiplying both sides of the equation by x and then solving the resulting linear equation. A more complicated attack was to instead multiply both sides of the equation by x^2 to produce a quadratic equation. Candidates could then still gain full marks by rejecting the solution $x = 0$ and correctly concluding that $x = \frac{4}{3}$.

(c) Most responses proceeded by splitting the part into two linear components. However, some candidates chose to square both sides of the equation and to deal instead with the resulting quadratic equation. Intuitive arguments based upon distances from $x = -1$ were rare but generally successful.

(d) Almost all candidates recognised that this part was a calculus question. Common errors were incorrect derivatives obtaining $y' = 4x - 3$ or $y' = 4x^3 - 3x$. The use of the derivative was also problematic with some responses attempting to calculate stationary points and others solving $4x^3 - 3 = -2$.

(e) Common errors in this part were the use of degrees rather than radians and the presentation of multiple solutions.

(f) A significant number of candidates did not make any attempt to solve an equation and simply evaluated $\ln 2$ correct to 4 decimal places. Most students who solved the equation to get $x = e^2$ went on to correctly evaluate their answer to 4 decimal places. Candidates are reminded that when dealing with questions demanding the presentation of an answer to a certain number of decimal places they should first write down their answer with a couple of extra decimal places and then write the answer to the requested number of places.

Question 2

(a) (i) Most candidates successfully used the product rule. The most common error was using $-\cos x$ as the derivative of $\sin x$. Candidates are reminded that the derivative of $\sin x$ can be obtained using the standard integral sheet available on the back of the examination paper.

(ii) The chain rule was the most popular method used for differentiation with only a small number of candidates choosing to expand the original expression. Common incorrect responses included $f'(x) = 2(e^x + 1)$, $2(e^x + 1)^2 \times e^x$ and $(e^x + 1) \times e^x$. Those candidates who chose the expansion method quite often incorrectly expanded, obtaining $f(x) = e^{x^2} + 2e^x + 1$.

(b) (i) The most common incorrect responses were $\dfrac{5^2}{2}$, $\ln 5$ and 0.

(ii) Candidates who showed setting out which included the first line of working – $\int 3(x-6)^{-2}dx$ — generally gave a final response that achieved full or part marks. The most popular incorrect response involved the use of logarithms in the primitive function, presumably because the function had a denominator. A few responses incorrectly attempted to find $\int (3x-18)^{-2}\,dx$, others used primitives involving inverse trigonometric functions, some differentiated and a few incorrectly expanded the denominator and followed this by an attempt to integrate term by term.

(iii) The better responses in this part involved a fully worked solution displaying four steps, firstly a preparation step involving rewriting $\sqrt{x}$ as $x^{\frac{1}{2}}$, followed by a primitive consisting of two terms, a substitution of correct limits into the primitive with clear indication of signs and brackets, and finally a careful computation of fractions. Candidates were more successful when they explicitly showed the substitution of their limits. Incorrect primitives of $x^{\frac{1}{2}}$ included $\dfrac{3}{2}x^{\frac{3}{2}}$ or $x^{\frac{3}{2}}$ and some candidates differentiated both terms. Other common errors resulted from the incorrect evaluation of terms with fractional exponents.

(c) Most candidates recognised the need to sum a series of four terms. However, many candidates incorrectly chose to use arithmetic or geometric summation formulae. In this part, full setting out including the use of brackets around the substitution of negative numbers $(-1)^1 \times 1^2 + (-1)^2 \times 2^2 + (-1)^3 \times 3^2 + (-1)^4 \times 4^2$, helped candidates to avoid careless errors involving the misuse of signs.

Question 3

(a) Many candidates used the formula $S_n = \frac{n}{2}(a+l)$. However, a number of candidates still preferred to use the formula $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ having to find the common difference using $T_n = a+(n-1)d$. Candidates using this approach often arrived at an incorrect value of *d*, or else found *d* correctly and then incorrectly stated the formula. There was minimal confusion with the notation for T_n and S_n. A significant number of students did not quote either formula correctly.

(b)(i) Some careless errors were made in rearranging the equation into the desired form and only a few candidates found the reciprocal of the gradient using $\frac{x_2 - x_1}{y_2 - y_1}$. A large number of candidates left the general equation with fractions, which made the working in the following part significantly harder.

(ii) Some candidates did not realise that this question required them to find the perpendicular distance. Common errors in using the formula included: quoting (or not quoting, but using) $\frac{|ax_1 + by_1 + c|}{\sqrt{x_1^2 + y_1^2}}$; not using absolute value signs in the perpendicular distance formula; changing the signs within the absolute value signs before adding; careless substitutions especially with $42 + -32$.

A significant number of candidates found the equation of *NP*, used that with the equation of *LM* to find *P*, and then used the distance formula. Many who used this approach received full marks but at the cost of completing more than a page of working. Others who were not successful wasted time using this approach and gave up half-way.

(iii) Most candidates did this part well if they had found a value for *NP* in (ii). A significant number of students could not do this question because they had no answer in part (ii), or forgot to answer it because of this or because they did not number the parts carefully. A significant number did not know the equation of a circle with centre not at the origin. Some diagrams were very small and often the region intended was not clearly shaded.

(c) The graphs were generally not drawn well. There was some uncertainty as to the region to be shaded. A number of candidates did not recognise that $y = 4 - x^2$ is a concave-down parabola. Some did not recognise it as a parabola.

(d) This part was done well by most candidates, with many achieving two or three marks. Candidates did this part in a variety of ways including $\frac{h}{3}\left(y_0 + y_n + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots)\right)$; or used three applications of $\frac{b-a}{6}\left(f(a) + 4\left(\frac{a+b}{2}\right) + f(b)\right)$ or $\frac{h}{3}\left(f(a) + 4\left(\frac{a+b}{2}\right) + f(b)\right)$; or used a table. Some candidates were unable to determine the value of *h* or were confused by the use of $\frac{b-a}{6}$. Candidates also applied Simpson’s Rule an incorrect number of times as a result of not reading the question. A minority of candidates applied the Trapezoidal Rule. Use of brackets was poor and often applied incorrectly, leading to mistakes.

Question 4

(a) Nearly all candidates realised that this part referred to an infinite geometric series. Common mistakes in those responses that did not achieve full marks were the use of an incorrect formula or an attempt to recalculate the common ratio from the first partial sums leading to $r = 1.9$.

(b) Most candidates realised that using the discriminant was a way to answer this part. Most candidates substituted into the equation of the discriminant but a significant number of these were then let down by poor basic algebra skills. In a small number of responses, candidates did not make the discriminant equal zero and so could not achieve full marks. Candidates who opted to use an alternative solution were often successful, with the relation between roots and coefficients being the most popular alternative.

(c) Most candidates were able to achieve some success in this part, but only a small percentage achieved full marks. This was often due to candidates not realising that there was a clear and intended connection between successive parts of the question. In better responses, candidates:

- drew a clear diagram at the top of the page
- used full and correct lettering notation for angles and sides
- included only the facts that were relevant to the question
- supplied clear and succinct reasons with their statements
- supplied clear conclusions to their proofs.

(i) The most direct way of proving similarity was by using equi-angularity and many candidates successfully completed this proof. For some responses taking this path, a common error was to state a connection between one pair of corresponding sides and then claim that one condition for similarity was two angles equal and one side in proportion. Candidates who opted to use the proportionality of two pairs of sides and equality of one angle were generally less successful.

(ii) Well answered by most candidates, but a significant number of successful responses were written as a connection between AP and AC, rather than as a ratio.

(iii) Successful responses hinged on stating that $AP = AC$, a direct consequence from part (ii), and candidates were rewarded for making this inference. Those candidates who then continued and proved the congruence of the two right-angled triangles that make up triangle AMC were generally successful. Other successful responses included the use of Pythagoras' Theorem and the property of the perpendicular bisector of the base of an isosceles triangle passing through the apex. Unsuccessful responses often involved circular arguments assuming the triangle was isosceles or trying to use the equality of angles.

(iv) A significant number of responses contained elaborate proofs that were not required of a part allocated one mark.

(v) Very few candidates were successful in this part. Responses that recognised the connection of this question to part (iv) and started by drawing a perpendicular from the base were often successful. Responses that started with an isosceles triangle were often unsuccessful as they indicated an ambiguity in the intent of the first dissection. Candidates are encouraged to use a ruler to draw diagrams and to take care in the way they indicate equal sides and angles.

Question 5

(a) (i) Most candidates used the formula $m_1 m_2 = -1$ to get the gradient of the normal and then used the point-gradient formula $y - y_1 = m(x - x_1)$ to obtain the equation of *BC*. A significant number of candidates made a simple calculation error. Candidates who showed correct working were rewarded with one mark.

(ii) Students who used the axes as the base and height of the triangle were more successful than those who attempted to use *AB* and *BC*.

(b) (i) Most candidates received full marks for this part.

(ii) A significant number of candidates did not understand this part and incorrectly applied the product theorem. Many candidates drew a tree diagram but did not write the correct probabilities.

(iii) Most candidates found this part challenging and did not apply the laws of probability. A common answer was $\frac{2}{3} \times 5 = \frac{10}{3}$. Candidates need to decide whether an answer is reasonable.

(c) (i) A significant number of candidates did not read this part carefully enough and used the area of a sector formula to calculate the other value of θ. Candidates who presented a logical argument received full marks.

(ii) Most candidates achieved the mark for this part by showing full working. A number of candidates used an incorrect formula.

(iii) A significant number of candidates used the cosine rule to find *AB* but failed to take the square root at the end. Some candidates correctly calculated the arc length and *AB* but did not find the perimeter of the minor segment.

Question 6

(a) This part of the question was generally done well, with most candidates beginning with $\pi \int y^2 dx$ and almost all of these successfully progressing to the correct answer. About half the candidates utilised the fact that the area required was symmetrical about the *y*-axis.

Those candidates who attempted this part but did not score full marks often did not correctly evaluate $\tan\frac{\pi}{3}$ or were careless with negative signs in the evaluation of either $\tan\left(-\frac{\pi}{3}\right)$ or $\left[\sqrt{3} - -\sqrt{3}\right]$.

(b) (i) A significant number of candidates seemed confused with the concept of half-life, not sure where to put $\frac{1}{2}$, or alternatively 2. Fewer candidates substituted 1600 in the correct position. Most correctly took the log of both sides of the exponential equation they had created. As in the previous part, a significant number of candidates were careless with negative signs, often ignoring them in order to achieve the positive value of *k* as stated in the question. A small number of candidates experienced difficulty dealing with the $A = 2A$ or $A = \frac{1}{2}A$ part of their equation.

(b) (ii) The responses to this part were very similar to responses in the previous part, although fewer candidates were successful here. Candidates are reminded that setting out their work in a neat, organised manner not only assists the marker to follow their working, but also helps them keep their ideas clear in their own mind.

(c) (i) This part of the question was either not attempted or was poorly done by many candidates. Many did not appear to see the relationship between the given gradients and the gradient of the curve at the point of contact of the two straight sections of track. Common errors included: finding the equation of the tangents using the given gradients and points; and creating equations which did not contain *a* or *b*, then trying to solve them using the parabola equation. Some candidates made careless mistakes with equations. For example, letting $x = 0$ in $2ax + b = 1.2$ produced $2a + b = 1.2$, or to progress from $2ax + b = 1.2$ to $ax + b = 0.6$, or to progress from $60a = -3$ to $a = -20$.

(c) (ii) Of the few candidates who were successful in the previous part, many were able to find the coordinates of the vertex of the parabola and the *y*-coordinate of the point on the parabola where *x* = 30 and hence calculate the required distance. Errors arose from candidates using 30 as the *y*-coordinate at *P*. Some candidates attempted to find the required distance using the distance formula or the perpendicular distance formula, but very few were successful.

Question 7

(a) (i) In the better responses, candidates were rigorous in the process of integrating twice and testing the initial conditions to evaluate the constants of integration. Other successful candidates differentiated the expression for displacement twice to obtain acceleration as well as testing the initial conditions. Candidates are reminded to make use of the standard integral when possible.

(ii) Almost all candidates realised that the particle was at rest when the velocity was zero. Better responses saw candidates solve the resulting equation by using their knowledge of equations reducible to a quadratic and then using log laws to obtain the appropriate time. Less successful candidates failed to see that the velocity equation obtained was in fact a quadratic. Candidates are reminded that there are no solutions for *t* in an equation that has e^{-t} equal to a negative number.

(iii) Most students realised that they had to use their answer from (ii) in the displacement equation to answer this part. This substitution was generally well done. However, the resulting evaluation indicated that more care was needed when using a calculator.

(b) (i) This part was not well done. Better responses resulted when candidates used the fact that the period was $\frac{2\pi}{n}$ or when they used a graph correctly.

(ii) Candidates with the most elegant solutions realised that the least value of $\sin\frac{\pi}{6}t$ was -1 and this occurred when $\frac{\pi}{6}t = \frac{3\pi}{2}$. Candidates who used a graph also quickly found the correct answers. Other candidates achieved success by using calculus to find the stationary points and then tested them to identify the appropriate minimum and its associated value. Many candidates who adopted this approach failed to test the stationary points to locate the minimum and used $t = 3$ instead of $t = 9$. A large number of candidates thought that low tide was at $t = 0$.

(iii) Most candidates were able to arrive at a point where they obtained $\sin\frac{\pi}{6}t \geq \frac{1}{2}$. From this point they often found only one solution, neglecting angles in other quadrants. Many candidates who correctly obtained $t = 1$ and $t = 5$ failed to convert this to an answer between 6 am and 10 am. Some candidates mistakenly interpreted the equation as linear.

Question 8

(a) Overall, many candidates did not have a deep and clear understanding of the relationship between a curve and its gradient function. Instead of looking at the value of $f'(x)$,the candidates were trying to describe the features of $f(x)$ and drew an incorrect graph.

(i) Most of the candidates were aware of the regions where $f'(x)$ was negative but they included the stationary points in their solution, writing $-1 \leq x \leq 3$. Some candidates gave the values of –1 and 3 for *x*, and many offered answers for where $f(x)$ was negative, which indicated poor comprehension of the question.

(ii) This part was poorly answered. Most candidates stated what happened to $f(x)$ rather than $f'(x)$ as *x* approached infinity. They were confused by the horizontal asymptote in the graph and stated that as *x* approached infinity, $f'(x)$ approached an asymptote or a constant *y* value.

(iii) Most candidates who attempted this part were able to gain a mark by drawing a concave-up parabola with intercepts at $x = -1$ and $x = 3$. Most candidates were unable to determine what happened to the graph as x gets smaller and also when x gets larger. Most graphs were poorly drawn with very little attention to accuracy, axes were not labelled and scales were poorly done. Few candidates were able to get full marks.

(b) The magnitude of the numbers that candidates needed to manipulate in this part proved a difficulty. Many of the errors could be explained by careless setting-out of working and very poor handwriting, especially where indices were involved.

(i) This part was well attempted by most candidates. Common errors were the poor understanding of the process of time payment (Amount owing = Balance + interest – payment) and incorrect calculation of the monthly interest rate.

(ii) Many candidates failed to set A_{288} to zero to set up the equation. Those that went on to solve the equation for M were able to score 2 or 3 marks depending on their computational accuracy. Many candidates showed inaccuracy in their use of algebraic skills, in particular when expanding brackets preceded by a negative sign. Incorrect geometric sum formula for the sum of n terms caused confusion for many. Candidates misinterpreted the value for n, which left a wide variety of answers that showed that many students did not really understand the concept that was being addressed.

(iii) Many candidates set up the equation correctly but did not go on with the algebraic manipulation to find n. Some candidates who found an expression for n used logarithms to find the correct answer. Most candidates did not proceed any further, showing a lack of knowledge in handling this type of evaluation involving factorising with powers and logarithms. Some candidates used trial and error to arrive at a solution.

(iv) Candidates who had perfect solutions in (ii) and (iii) made poor attempts here as they either misunderstood the question or used incorrect values in their calculations. Most candidates did not attempt this part as they had not completed part (iii). Some candidates, who had little idea as to what was required to get the correct answer, went back to incorrectly work through the geometric series again.

Question 9

(a) Common errors in this part included: not treating the two raffles as independent events; not combining the two independent events; or writing down decimals with no accompanying explanations.

(b) (i) The most common errors in this part were: using the cost of $1000 for both the shore and underwater component of the cost; using $1000 for the underwater component and $2600 for the shore component; and not adding together the two components of the cost.

(ii) Most candidates calculated the distance QS using Pythagoras' Theorem, although some successfully used the cosine rule. The most common errors in this part were arithmetic errors, for example $\sqrt{5^2+3^2}=\sqrt{36}=6$, and using $1000 as the cost.

(iii) Better responses to this part derived the expression for C in a logical sequence, explaining each step. The most common error in this part was to begin with the given result merely expanded without any explanations.

(iv) Common errors in this part included: incorrect use of the chain rule; algebraic errors, including simplifying $\sqrt{x^2+9}$ to $x+3$; not testing the nature of the stationary point at all; not finding the second derivative correctly if using the second derivative test; and, if using the first derivative test, just writing \ _ / in the table without testing any values.

(v) Better responses to this part substituted 1.1 for 2.6 in their working for part (iv). Common errors included: not using calculus at all but rather calculating $C(0)$, $C(5)$ and $C\left(\frac{5}{4}\right)$ for the revised expression for the cost C and taking the smallest of these; successfully calculating that the new minimum occurs at $x=\frac{30}{\sqrt{21}}$ and claiming this to be the new path; or being confused that the new minimum occurred at a value greater than 5 and crossing out work thinking it to be incorrect.

Question 10

(a) Candidates are reminded that when a question asks them to show or prove something, then it requires that clear and logical working be shown. Candidates whose conclusions lead to a contradiction are reminded that this signals that an error has been made and that returning to check working could locate the error. Some candidates had a problem working with fractions.

Many candidates obtained the correct first derivative $f'(x) = 1 - x + x^2$, but in an attempt to rewrite the expression and start with the term x^2 the incorrect use of the minus sign led to many mistakes.

Candidates are reminded that even if a quadratic equation cannot be factorised it may still have real roots.

Most of the correct responses involved the use of the discriminant to argue that the quadratic equation $f'(x) = 0$ has no real solutions when the discriminant is negative.

A few candidates wrote the derivative in the form $\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$ and then successfully argued that $f'(x) \neq 0$.

(b) Most candidates found the point of inflection by setting $f''(x) = 0$.

(c) When cancelling terms in an algebraic expression, they can be crossed out but this should not be done so heavily that the terms are no longer legible.

The statements in parts (i) and (ii) cannot be proved by the substitution of several values of x into both sides.

The fact that 'Let $f(x) =$ ...' occurs before the separate parts of this question implies that this function will apply throughout the entire question. A common error was to take $f(x) = \dfrac{x^3}{1+x}$ in part (c) (ii) and also later in part (f).

Many candidates differentiated $\ln(1+x)$ correctly.

(d) Notice should be taken of what has been developed in previous parts of a question, particularly the significance of the fact that $f'(x) > g'(x)$. Also it was important to realise that both graphs pass through the origin. Many incorrect responses did not have the graph of $f(x)$ above the graph of $g(x)$ for $x > 0$.

If more than one graph is shown on the same set of axes then each must be labelled. Diagrams need to be large and clear.

(e) Responses to this part became messy when some candidates developed expressions with incorrect use of brackets. Ironically, many candidates had more success in differentiating $(1+x)\ln(1+x)$ by using the product rule than in differentiating the simpler expression $-(1+x)$.

(f) Candidates are encouraged to make full use of the table of standard integrals included at the end of the examination paper. Many candidates made an error by writing $\int \ln(1-x)dx = \frac{1}{1+x}$ instead of using the link in part (e).

Some candidates correctly managed to evaluate $\int_0^1 \ln(1-x)dx$ by using the area between the curve and the y-axis.