

- (ii) Candidates' responses varied. Better candidates achieved the required result. The early introduction of a constant of integration, for example $x = \frac{1}{a} \ln\left(\frac{y}{1-y}\right) + k$ or

$ax = \ln\left(\frac{y}{1-y}\right) + k$, induced algebraic errors while progressing to the required function

$$y = \frac{1}{ke^{-ax} + 1}.$$

- (iii) Many candidates were successful in finding only that $k = 9$, but were challenged by the other parts of (c). Many interpreted the question to imply that $\frac{1}{2} \times \frac{1}{10} = \frac{1}{k+1}$ or $2 \times 10 = k + 1$.
- (iv) Many candidates interpreted this question correctly by using the result found in part (i) by stating it had a maximum gradient or a point of inflexion at $y = \frac{1}{2}$.
- (v) The responses indicated that the candidates were very challenged by this part. Many candidates who had answered part (iv) correctly did not use the information to advantage in sketching the function. Many had their point of inflexion at $x = 0$.

Question 6

- (a) (i) Many methods were used to show this result. Successful responses included a variety of similar triangle solutions (with diagrams to explain which triangles were being compared), areas of trapezia, or a consideration of the linear relationship between s and x . Partly successful responses approached the problem in a similar fashion did not complete the calculation. An example using similar triangles was to divide a vertical cross-section of the frustum into a parallelogram and a triangle, or to add on a triangle to create a parallelogram, for example using one of the following:



- (ii) Successful responses to this part presented an integral of the area of the cross-section from $x = 0$ to $x = h$. As part (i) supplied the cross-sectional area, this approach was quite common. Less than successful responses made a variety of errors when integrating or expanding $\left(a - \frac{a-b}{h}x\right)^2$, or by using incorrect limits, for example from a to b , or by integrating the side length s rather than the area s^2 .
- (b) Successful responses presented a first step to prove the result to be true for the two initial values of n . Assuming the result was true for $n = k - 1$ and $n = k$ provided the necessary link to prove the general result. Those who completed this were able to first factorise $2(1 + \sqrt{2})^k + (1 + \sqrt{2})^{k-1}$ and $2(1 - \sqrt{2})^k + (1 - \sqrt{2})^{k-1}$. Responses that started with $2(1 + \sqrt{2})^k + (1 - \sqrt{2})^{k-1}$ in an attempt to work towards $2a_k + a_{k-1}$ were generally less successful.

- (c) (i) The binomial expansion was generally well done.
- (ii) Most responses successfully presented an expansion using De Moivre's theorem and most of these went on to confirm the desired result by first equating the imaginary parts and then correctly replacing $\cos^2 \theta$ with $1 - \sin^2 \theta$.
- (iii) Successful responses recognised and used the result in part (ii) and noted that $\sin 5\theta = 1$ when $\theta = \frac{\pi}{10}$. Hence, by subtracting 1 from both sides, the given polynomial in x was found.
- (iv) This part was quite well done, usually by long division. The polynomial $p(x)$ was also found by other means, including by inspection. The long division process was, however, prone to errors.
- (v) Expanding the first few terms of $(4x^2 + ax + 1)^2$ and equating the coefficients of x^3 was the most common successful approach. Responses that equated the coefficients of x^2 found two possible values for a . Checking the coefficients of x^3 , the correct response was generally found.
- (vi) The responses that recognised the need to solve the equation $4x^2 + 2x + 1 = 0$, together with the fact that $\sin \frac{\pi}{10} > 0$, were mostly successful.

Question 7

- (a) (i) It should be noted that to prove that two triangles are similar, it is sufficient to prove two angles are equal in the relevant triangles (which most candidates did) but a significant number wasted time by trying to show that the third angle was also equal in all triangles.
- (ii) The best attempts found relevant equivalent ratios and substituted into the required expression. Many attempts were far less efficient and included lists of various ratios, not all of which were relevant.
- (iii) Most candidates who realised that all the diagonals of the regular pentagon were equal to x successfully answered this part. Many candidates referred to sides or diagonals in such terms AD or BC . However, as this notation was neither explained on the given diagram nor in the candidate's working, the markers were unsure which distances the candidates were referring to.
- (b) This part was successfully done by most candidates. Some attempted to answer this question without actually drawing the graph. This approach required a much more detailed and precise explanation in order to be rewarded.
- (c) (i) Some candidates made errors using index laws and others had trouble solving equations such as $x^{n-2} = x^{n-1}$ or $x^{n-2} = 0$.
- (ii) Although this part was successfully done by most candidates, some only showed that $P'(0) = 0$ without showing that $P(1) = 0$. Some substituted $n = 1$ instead of $x = 1$.

- (iii) Most candidates gave a sketch of $y = P(x)$ but many failed to realise that $(0,1)$ is a maximum stationary point. Many did not state that there is another real zero because the graph crosses the x -axis at a point other than $x = 1$. Many did not draw a graph that was consistent with all the facts found in the previous two parts of this question.
- (iv) Most candidates who attempted this part did not appear to realise that they needed to show that $P(-1) < 0$ and $P(-0.5) \geq 0$. Also, many did not use the fact that $(-1)^n = -1$ and $(-1)^{n-1} = 1$, since n is odd. Some of the few candidates who did make progress with this part failed to get an expression in a form that allowed them to use part (b).
- (v) Most candidates who attempted this part failed to realise that the polynomial had 5 zeros, three of which are 1, 1 and a , and that the other two are complex conjugates. A significant number said that the product of the zeros was 0.25, instead of the correct -0.25 .

Question 8

A large number of candidates did not attempt this question. What appeared to be rushed work indicates that some candidates left themselves little time for this question. Candidates who made their application of integration by parts very explicit fared best in getting the derivatives, primitives and algebra (including signs) correct.

- (a) Many candidates replaced a factor of $\cos^2 x$ with $1 - \sin^2 x$ immediately and then could not go further or had a more difficult solution. Some used the double angle formula for a $\cos^2 x$ or $\sin^2 x$ factor but in most cases they failed to reach a correct solution. Algebraic errors were common, as were subscript errors, typically A_{n-2} for A_{n-1} .
- (b) This part was generally well done.
- (c) A good number of attempts did not make a correct choice of u and v^1 using integration by parts as a way of introducing a factor of x^2 . Some candidates persisted with poor choices such as $v^1 = \cos 2x$ or $x \sin x$, that generally went nowhere. Poor setting out or rushed work often led to algebraic errors. A correct solution starting with B_n was very rare.
- (d) Many candidates tried this part, but with mixed results. Some quickly got the result in a line or two; others struggled with up to a page of algebra.
- (e) Some candidates did not see the telescoping sum.
- (f) This part was rarely attempted, but in those cases the candidate usually made substantial progress. Some candidates tried to argue that $\cos x = \sin\left(\frac{\pi}{2} - x\right) \geq \frac{2}{\pi}\left(\frac{\pi}{2} - x\right) = 1 - \frac{2x}{\pi}$, rather than $\cos^2 x = 1 - \sin^2 x \leq 1 - \left(\frac{2x}{\pi}\right)^2$, but this leads to a inequality different from the one desired.
- (g) This part was generally poorly done in the few cases it was attempted. The most common error was the calculation of the primitive in the attempt at integration by parts, incorrectly

finding the primitive $-\frac{\pi^2}{8x(n+1)}\left(1-\frac{4x^2}{\pi^2}\right)^{n+1}$ from the chosen integrand $\left(1-\frac{4x^2}{\pi^2}\right)^n$.

Some candidates tried expressing the factor x^2 in terms of $1-\frac{4x^2}{\pi^2}$ but this did not lead to a solution. A small number took the easier route of correctly working from right to left.

- (h) Those candidates who attempted this part generally made a correct change of variables in this integral, but very few could provide the reason for the inequality with A_n .
- (i) This part was rarely attempted. It required using the formula from part (e) and the inequality from part (h).
- (j) Many candidates wrote down the correct answer $\frac{\pi^2}{6}$, with or without justification, even if the rest of this question was poorly done.