# 2011 HSC Notes from the Marking Centre – Mathematics

## Contents



### Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics. It contains comments on candidate responses to the 2011 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2011 Higher School Certificate examination, the marking guidelines and other support documents developed by the Board of Studies to assist in the teaching and learning of Mathematics.

Candidates are reminded of the importance of reading the question carefully, looking for links between parts of a question, solving equations carefully, computing accurately, labelling tables and graphs appropriately, and setting out their work clearly.

In answering parts of questions, candidates should state the relevant formulas and the information they use to substitute into the formulas. Working is expected to be shown and, in general, candidates who show working make fewer mistakes. When mistakes are made, marks may be awarded for the working shown. Any rough working should be included in the answer booklet for that question. Candidates are also reminded that a table of standard integrals is provided.

### Question 1

(a) Many candidates wrote down the correct answer without showing any working. Better responses showed the full calculator display followed by the correct rounding to 4 significant figures. By showing this working, candidates still show that they can round to 4 significant figures even if they made an error in their calculator work.

The most common errors included, calculating $\sqrt[3]{\frac{651}{4}\times\pi}$ , rounding to 3 significant figures and rounding to 4 decimal places.

(b) The most common errors included incorrect factorisation, for example, $\frac{n^2-25}{n-5}=\frac{(n-5)^2}{(n-5)}=n-5$ and incorrect cancelling, such as $\frac{\cancel{n^2}-\cancel{25}}{\cancel{n}-\cancel{5}}=n-5$.

(c) Nearly as many candidates approached this question by taking logarithms of both sides compared to writing 32 as $2^5$ and equating indices. A number of candidates misread the question as $2^{x+1}=32$. Many candidates incorrectly manipulated the expression involving logarithms to obtain $x=\frac{\log(32)}{2\log(2)}-1$ or incorrectly divided by 2 to obtain $2x+1=16$. Some candidates did not use the same base for the logarithm on each side of the equation.

(d) Many candidates did not use the chain rule and obtained $\frac{d}{dx}(\ln(5x+2))=\frac{1}{5x+2}$. Some treated the function as a product by inappropriately using $u=\ln$, $v=5x+2$ and incorrectly obtained $u'=\frac{1}{x}$ .

(e) Better responses to this part showed all steps in finding the answer. Some candidates divided by a negative number incorrectly, simplifying $-3x\le 6$ to either $x\ge 2$ or $x\le -2$. Others treated the problem as if it involved absolute values, solving $-8\le 2-3x\le 8$.

(f) In better responses, the working was carefully set out. In many responses, the square root signs were 'lost' during the working. The most common errors included not multiplying by the conjugate, eg multiplying by $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and incorrectly expanding the denominator to obtain either $5+3$ or $5-9$. Some did not give the simplest form of the answer, eg $\frac{4(\sqrt{5}+\sqrt{3})}{2}$ rather than $2(\sqrt{5}+\sqrt{3})$.

(g) Nearly all candidates answered this part successfully. The only common mistake was to calculate $800\div 0.02$.

**Question 2**

(a) This part was generally done well. The main error was to incorrectly quote the rules for the sum and product of the quadratic roots. When this occurred, the mark for part (iii) could still be obtained for evaluating $\frac{\alpha+\beta}{\alpha\beta}$ from the previous answers. In some responses the irrational roots were calculated using the quadratic formula and full marks were possible at the expense of time, working and the likelihood of errors. There were a number of non-attempts for part (iii) and quite a few incorrect attempts to add the fractions $\frac{1}{\alpha}+\frac{1}{\beta}$ .

(b) This part was generally done well with the majority using exact values in radians and finding results in the correct quadrants. Relatively few responses were given in degrees only. A small number worked in radians but gave decimal approximations. The better responses used a circle diagram to work out the quadrants, used 60º or $\frac{\pi}{3}$ as a reference angle and correctly simplified the resulting sums of fractions $\pi + \frac{\pi}{3}$ and $2\pi - \frac{\pi}{3}$ . The most common problems involved converting from degrees to radians and giving only one solution. In some responses with –60º, the negative sign proved a problem.

(c) Nearly all candidates recognised this as a calculus question and showed logical working that demonstrated a good understanding of the required steps. This allowed for part marks for the occasional imperfect responses. There were errors in differentiating, with $4(2x+1)^3$ or $8x(2x+1)^3$ being the usual incorrect answers, but often the rest of the working followed correctly. Sometimes notation was poor and lack of parentheses resulted in the wrong gradient or point. Candidates are encouraged to show clear substitutions to avoid careless errors. Several candidates substituted $x=0$ and either used $(0,1)$ or $(-1,-1)$ in their equation of the line. In better responses, candidates clearly showed the derivative, the gradient *m*, the point and finally the equation of the line. A small number of candidates provided only the gradient of the tangent rather than the equation of the tangent (stopping at $m=-8$ ). Some also correctly evaluated $f'(-1)=-8$ then used this as the *y* value of the point rather than as the gradient. Candidates need to take care when copying the *x* value into the writing booklet as some used $x=1$ instead of $x=-1$.

(d) This part of the question was completed successfully by most candidates. Many were assisted by using an organisation area for $u$ , $u'$ , $v$ and $v'$ . Few quoted the product rule but most were able to write the derivative expressions correctly. The occasional errors were in having an inappropriate negative sign in the rule, or by having the derivative of $e^x$ as $xe^x$.

(e) Responses that brought the coefficient $\frac{1}{3}$ to the front of the integral were generally more successful. The most common error was to integrate $3x^{-2}$ instead of $\frac{1}{3}x^{-2}$. Another common error was to interpret this as a logarithm integral. Also, the use of the integral sign was poor, being written after the integration had been performed or being left out completely.

## Question 3

(a) (i) In the majority of responses, candidates recognised that this question involved an arithmetic sequence and either substituted into the formula correctly or worked out the pattern for obtaining the cost of the 25th floor. The most common errors were to use the incorrect formula and to find the sum of a series, either arithmetic or geometric.

(ii) In better responses, candidates stated the formula for the sum and showed the substitution before any calculation was performed. Many incorrectly found the cost of the 110th floor rather than the total cost of all 110 floors.

In both subparts, many who substituted correctly into the relevant formula were then unable to correctly evaluate the final answer.

(b) In responses that included an accurate diagram, the point (3, –1) as the vertex was easily obtained. A significant number correctly stated that $x = 3$, but found an incorrect $y$ value. Errors resulted from not knowing that the vertex is halfway between the focus and directrix or from a lack of basic cartesian plane knowledge, with a common incorrect solution being (–1, 3).

(c) (i) This part was answered well with the majority of responses having the substitution of $x = 0$ into $l_1$ then solving for $y$. Some expressed $l_1$ in the form $y = mx + b$ and stated that the $y$-intercept was 3.

(ii) In better responses, candidates rearranged the equations for $l_1$ and $l_2$ into the gradient-intercept form, stated the gradients and showed that their product was $-1$. Many who attempted to use the general form of the equation and the fact that the gradient is given by $m = -\frac{a}{b}$, made errors in stating this formula and hence obtained incorrect gradients. A time-consuming strategy used by many candidates was to find the point of intersection $E$ then the gradients of $OE$ and $BE$. Candidates should use and be familiar with correct mathematical terminology, particularly that the reciprocal is not an 'inverse' or 'flip'.

(iii) Most responses included the perpendicular distance formula with correct substitutions to obtain the required result. The most common error was to substitute $+12$ instead of $-12$ into the formula. Many who found the coordinates of $E$ then the distance from $E$ to the origin, made some errors.

(iv) The better responses used the lengths of two sides known from parts (i) and (iii) and applied Pythagoras' Theorem to correctly to find $BE$. Several unnecessarily recalculated $OE$. Many used the distance formula and the points $B$ and $E$ to calculate the length of $BE$.

(v) This part was done well. In most responses, answers to parts (iii) and (iv) were used to find the area of the triangle.

**Question 4**

(a) This question was done well, with most candidates recognising that they were required to use the quotient rule. Many were assisted by writing $u$, $u'$, $v$ and $v'$. A small number quoted the quotient rule incorrectly. Some also differentiated $\sin x$ incorrectly to arrive at $-\cos x$. Responses in which the product rule was used to differentiate $x(\sin x)^{-1}$ were rarely fully successful, the most common error being an incorrect differentiation of $(\sin x)^{-1}$.

(b) Most candidates recognised the primitive would be a log function, but a significant number used an incorrect constant or integrated to get $\ln(5x)$. Those who integrated correctly usually went on to evaluate $\log e$ and $\log e^3$ correctly.

(c) The most common error was to substitute $x = -1$ into $\frac{dy}{dx} = 6x - 2$ then find the equation of the tangent to the curve, instead of finding the equation of the curve itself. Some integrated correctly, but then did not substitute the point (–1, 4) correctly to evaluate the constant of integration. Many substituted 0 for $y$.

(d) (i) The most common error was to multiply by the incorrect derivative (usually $2x$ instead of $-2x$) when using the chain rule. In a small number of cases, one was added to the power when differentiating, instead of being subtracted and, in some, the fractional index was a problem.

(ii) In many responses, candidates did not connect parts (i) and (ii) and attempted to integrate incorrectly using the reverse of the chain rule. Responses in which candidates were able to isolate their integral using part (i) were usually successful.

(e) This part was more challenging. The most common errors were to give the domain and range as the inequalities or to find the area between the two curves. There were a significant number of non-attempts for this question.

**Question 5**

(a) With the exception of part (i), this was a challenging question. Many responses used an arithmetic series. Common errors in all parts were to mix the formulae for geometric term and geometric sum, or to misquote formulae. Listing all terms was common and many successfully found all answers this way, but at a large cost in time.

(i) In better responses, the formula for $T_n$ was used with the substitution clearly shown.

(ii) In better responses, again, the formula for $T_n$ was used to form an equation or inequality that was successfully solved. 10 million was often incorrectly interpreted, commonly as 1 000 000. Many elementary errors were made when solving the equation and many could not correctly interpret the inequality to obtain day 20. Success from guess-and-check was common, but candidates are reminded to show the guesses and not just give the final answer.

(iii) In better responses the formula for the sum of the series was used to calculate the total membership after 12 days, followed by the calculation of the money earned. Common errors included finding the amount earned for the $12^{th}$ day, using incorrect values of $r$ (usually 0.5), calculating an answer in cents but calling it dollars or not converting to dollars at all.

(b) Better responses included a probability tree in a large (half page) diagram, followed by successful interpretations to obtain correct answers to all parts.

(i) This part was done well with most candidates simply stating the correct answer.

(ii) In better responses, candidates clearly stated what probability they were calculating, showed the calculation and correctly evaluated it. Common errors were related to not recognising that there were only two yellow shirts. A probability of zero for the third day was frequently ignored or treated as a probability of 1. Numerical expressions were not simplified correctly, further highlighting the need to show working and to avoid giving bald answers.

(iii) This part was challenging. In most responses, candidates incorrectly assumed they needed the complement of their answer to part (ii), and many did not correctly interpret the meaning of 'consecutive days'. Candidates are encouraged to clearly write in words or symbols exactly what probability they are trying to find, then show the calculations needed.

(c) The majority of responses correctly used the 5 function-values, often in a table, to find the approximate distance. Common errors included the use of function notation in the quoted formula with incorrect substitutions and the incorrect calculation of the value of $h$ using a formula. While two separate applications of the formula using 3 function values was correctly done by some, it frequently led to errors. Incorrect weighting and missing brackets also resulted in errors. In some responses there was confusion about the relationship between velocity, displacement, integration and Simpson's rule, resulting in Simpson's rule not being applied. Numerical errors were also common, again highlighting the need to show full substitutions into a formula before any evaluation.

## Question 6

(a) This part instructed candidates to draw a diagram and the majority of candidates did so. The diagram allowed for checking the candidate's working.

(i) Very few responses treated $\angle CDE$ as an internal angle of a regular pentagon. Many completed this question using the formula for the sum of external angles. Common errors included misinterpreting the question, with many candidates finding the incorrect angle, and using an incorrect formula/method to find the angle.

(ii) There were a number of non-attempts for this part. In better responses, candidates drew and labelled their diagram, showing the calculated angle sizes. In a large number of these cases there was no supporting statement and/or geometric reason. Common errors included proving that other irrelevant triangles were isosceles and accepting obviously incorrect answers, such as two obtuse angles in a triangle.

(b) This was a challenging question. In responses with correct substitution into $PA^2 + PB^2 = 40$, simple algebraic errors occurred in expanding and simplifying the expression, as well as in completing the square to find the centre and radius of the circle.

(c) (i) Generally done well.

(ii) A common error was $-2\sin x$ as the primitive, indicating that the table of standard integrals was not consulted.

(iii) Generally done well.

(iv) Many responses showed three separate areas using integration rather than connecting parts (ii), (iii) and (iv). A number of candidates gave the area as zero.

(v) A common error was to find the area between $x = \frac{\pi}{2}$ and $x = 2\pi$ rather than the value of the integral over this domain, demonstrating a poor understanding of the difference between finding an area and evaluating an integral. The instruction 'Using the parts above' was completely overlooked by many.

**Question 7**

(a) This part was generally done well. Transcription errors were common, for example $x^2 - 3x + 2$, $x^3 - 3x^2 + 2$ and $x^3 + 3x + 2$. Such errors may result in the necessary skills not being demonstrated.

(i) Most candidates attempted to solve $f'(x) = 0$, with a significant number giving $x = 1$ or $x = 0, 1$ or $x = 0, 1, -1$ or $x = \sqrt{3}, \sqrt{-3}$. Many candidates used tables of values that were not labelled adequately, if at all. Numerical errors made tables difficult to interpret and points impossible to graph meaningfully.

(ii) The better responses used the results from (i) to produce a neat, well-labelled sketch that clearly showed the *y*-intercept and other important points. A number of candidates plotted points from a table of values and did not use answers to part (i).

(b) In better responses, candidates linked the particle's initial velocity of 0 with its positive acceleration to explain the movement in the positive direction then linked the limiting velocity to sketch velocity against time.

(i) Most candidates substituted $t = 0$ to show that $\dot{x} = 0$. Some solved $\dot{x} = 0$ to show that $t = 0$ and then used the fact that the particle does not stop again to help answer part (iii).

(ii) In many responses candidates differentiated incorrectly, giving $-16e^{-2t}$ and crossing out the negative sign in order to answer the question. Others gave $e^{-3t}$ or $e^{-2t-1}$.

(iii) In better responses, candidates described the motion by linking the concepts in parts (i) and (ii). Others successfully solved $8-8e^{-2t}>0$ to show that $t>0$, while some struggled with negative expressions and negative powers to show that $t<0$. In many responses candidates claimed that positive acceleration indicates positive velocity. Many candidates integrated to find $x$ and claimed that positive displacement indicated movement in the positive direction.

(iv) This part was done well especially when $8-8e^{-2t}$ was written as $8-\dfrac{8}{e^{2t}}$.

A significant number relied on substituting values of $t$ and observing the trend.

(v) In better responses, candidates clearly showed the curve increasing from the origin, concave down and approaching a clearly-drawn asymptote. Others resorted to completing a table of values and plotting points, rather than using their previous responses.

## Question 8

(a) (i) Generally this part was done very well. Common errors included difficulty in rearranging the cosine rule to obtain the result, using the cosine rule starting with $x^2$ on the left but still using the 60º angle, using an incorrect version of the cosine rule (including using sine not cosine in the formula) or using the incorrect value of $\cos 60^0$.

(ii) This part was done reasonably well. Common errors included the incorrect application of the quadratic formula (including quoting an incorrect formula), difficulty in completing the algebra and arithmetic involved in solving a quadratic equation (including inability to correctly simplify the expression inside the radical and not dividing by the denominator correctly) or attempting to factorise the given quadratic equation and determining two incorrect factors.

A number of candidates did not attempt to solve the quadratic equation in part (i), rather they approached the solution either by using the sine rule to get the value of angle *LSP* then using the cosine rule or the sine rule again to get the value of *x,* or by drawing a perpendicular from *S* to *LP* and finding the values of sections of *LP* using a combination of trigonometry and Pythagoras' theorem.

(b) (i) Common errors included the application of an incorrect volume formula (commonly confusing the axis of rotation) and incorrect evaluation of the definite integral (commonly using incorrect limits).

(ii) Many candidates were not able to complete all the steps required to correctly determine the ratio. Common errors included not being able to determine the radius of the cylinder, and not correctly stating the formula of the volume of a cylinder.

(c) (i) In better responses, candidates were able to establish the series or recognise a superannuation problem and substitute into a formula for the sum of *n* terms. Common errors included introducing an extra \$100 term, interpreting the value for

(ii) A small number of candidates integrated $t$, then easily calculated the correct answer of
8 litres. However, many attempted to integrate the given rates for $A$ and $B$ (most being unsuccessful in integrating $A$ ) rather than using the result from part (i) to answer part (ii). The majority of candidates substituted $t = 4$ into the original rates.

(c) In the better responses, candidates drew the given graph and the graph of the derivative below it. Many candidates were able to show or state that the graph cuts the x-axis at $x = 1$. Some misinterpreted the question and thought they should clearly indicate the features of the given graph rather than the graphs they had drawn. Many candidates did not clearly indicate on their graph what was happening at important points, such as at $x = 3$ and as $x \to \infty$.

(d) (i) This part was generally done well. A majority of candidates realised the need to multiply top and bottom by $\sqrt{n} - \sqrt{n+1}$. However, many then had difficulty with the algebra involved in simplifying the denominator. Most recognised a difference of two squares, but commonly wrote $\left(\sqrt{n} + \sqrt{n+1}\right)\left(\sqrt{n} - \sqrt{n+1}\right) = n - n + 1$.

Common features were the omission of the initial working line for rationalising the denominator, illegible writing, poor use of the surd sign such as $\sqrt{n} + 1$ for $\sqrt{n+1}$, and not putting brackets around the terms in the denominator when squaring them.

(ii) Many candidates did not connect parts (i) and (ii). Those who did easily completed the solution. A common error was to suppose the series to be either arithmetic or geometric and to spend time trying to test the series to find either a common difference or common ratio. A significant number of candidates who substituted the values for $n$ into the correct expression from part (i) did not recognise the collapsing sum.

## Question 10

(a) (i) Common errors included incorrect calculator use and assuming $e^{11} \times 10^{-12}$ to be correctly expressed in scientific notation.

(ii) Most candidates showed a clear understanding of this question and correctly substituted into the given equation. The common errors involved the incorrect manipulation of indices and the misuse of logarithms. A number of candidates misinterpreted the meaning of the word 'maximum' and attempted to solve a differential equation.

(iii) This part was challenging. The popular methods used to find the increase in loudness involved either an algebraic approach or the use of results from parts (i) and (ii). Some candidates introduced other values for $I$, doubled the intensity and compared the resulting expressions for loudness. A simple solution involved the use of the exponential growth model $I = I_0 e^{0.1 \times L}$ with $I = 2I_0$. In many responses, candidates incorrectly doubled both sides of the given equation rather than only doubling the intensity. A few responses used squaring instead of doubling and others doubled the loudness rather than the intensity. A number of candidates misinterpreted the question and found an expression for the increased loudness.

(b) Candidates are reminded that when a question asks to 'show' a result, they are required to demonstrate clear and logical working. When asked to 'explain', candidates should support their answer with a mathematical argument.

(i) The majority of responses included the correct formula.

(ii) The most popular and succinct approach involved stating the formula for the area of a sector and substituting $\theta$ in terms of $r$ and $P$ from part (i). Those who started with the given result $A = \frac{1}{2}\text{Pr} - r^2$ and substituted the result from part (i) quite often found the resulting algebraic manipulation difficult.

(iii) The majority of candidates recognised the need to use calculus in this question. The most common and successful method was to solve $\frac{dA}{dr} = 0$ for $r$, then test by the second derivative. Candidates who used the first derivative test often omitted or struggled to find first derivative values for $r < \frac{P}{4}$ and $r > \frac{P}{4}$. A number of candidates appeared to ignore the given direction involving $r$ and $P$ and tried to maximise an area expressed in terms of $\theta$ and either $P$ or $r$. This involved rigorous algebra.

(iv) Many candidates struggled to determine $\theta$ correctly. A significant number used calculus for a second time and maximised the expression for area in terms of $\theta$.

(v) This was a challenging question, with very few responses demonstrating a quality argument. Many candidates manipulated the given result and produced equations or inequations to support the situation. However, they often did not validate their findings. Those who commenced with a restriction on $\theta$, $A$ or $P$ were generally much more successful. A common and succinct method was to state the domain $0 < \theta < 2\pi$ for a sector to exist and use the expression for $\theta$ from part (i). Many explanations lacked reasoning and some candidates presented only a circular argument involving a suggested constraint.