2012 HSC Notes from the Marking Centre – Mathematics Extension 1

Contents

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course in Mathematics Extension 1. It contains comments on candidate responses to the 2012 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2012 Higher School Certificate examination, the marking guidelines and other support documents developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 1.

General comments

The instructions indicate that relevant mathematical reasoning and/or calculations should be included in the responses for Questions 11–14. Candidates are reminded that where a question is worth several marks, full marks may not be awarded for an answer, even if the answer given is correct, if no working is shown.

This is because mathematical communication and reasoning are included in the objectives and outcomes assessed by the examination.

Candidates are advised to show all their working so that marks can be awarded for some correct steps towards their answer. A simple example is when candidates have to round their answer to a certain degree of accuracy. Candidates should always write their calculator display before rounding their answer. They should only round their answer in the last step of working, not in an earlier step. Markers can then see that candidates have rounded correctly, even if the answer is not correct.

Question 11

a) In almost all responses, candidates recognised that using the table of standard integrals was the most suitable method to find the primitive. In better responses, candidates also applied appropriate substitutions leading to the correct solution. In some weaker responses, candidates inappropriately used degrees, or did not handle the co-efficient correctly.

b) In better responses, candidates recognised and used the product rule correctly. In weaker responses, candidates seemed to be mostly confused by the $\tan x$ function, given that the previous question involved an inverse tan. In many weaker responses, candidates used some rather cumbersome algebra/identity processes unnecessarily, having achieved a correct answer in their first line of working.

c) Many candidates recognised the domain restriction and wrote down $x \neq 3$. By far the most common method used to establish the critical points was to multiply both sides by the square of the denominator. In some responses, candidates performed some rather basic algebraic processes to arrive at critical values of 3 and 6. In better responses, the values in each region were tested or diagrams were drawn to verify the solution set.

d) Most candidates correctly found the correct derivative of the given substitution and/or changed the limits. In better responses, candidates correctly rearranged the integrand using the substitutions. In weaker responses, candidates made errors when trying to establish the integrand following the substitution. Some candidates handled the negative arising from $\frac{du}{dx}$ incorrectly.

e) Many candidates knew that binomial coefficients were needed to complete this part, but many were confused about whether to add them or multiply them.

f) (i) In better responses, candidates clearly found the position of the constant term and then the value of the constant term. In some weaker responses, candidates did not notice the negative. Some candidates attempted to write out the whole sequence, but this approach was mostly unrewarding.

(ii) Many candidates had difficulty with interpreting the phrase 'non-zero constant' or in referring to *n* and *k* as integers. There were some rather unusual algebraic statements. Overall, candidates found this part challenging.

Question 12

(a) Most candidates made reasonable progress in this induction proof. Almost all showed that $P(1)$ was true and wrote statements for $P(k)$ and $P(k+1)$. However, there were some transcription errors between statements. Most candidates used their assumption to complete a proof. Candidates who demonstrated better algebraic skills established $P(k+1)$ correctly. Candidates who followed standard methods often completed efficient and correct proofs. While many candidates could write a statement like $2^{3k} - 3^k = 5M$, they did not state that M was an integer. Some candidates used unorthodox mathematical notations, such as $2^{3k} - 3^k / 5$ or $2^{3k} - 3^k = \frac{P}{5}$ which do not demonstrate clear understanding. In some weaker responses, candidates substituted twice which made the task more complicated. A handful of candidates began with the $P(k)$ statement and built it up to arrive correctly at the $P(k+1)$ statement, in an unusual but valid approach.

(b) (i) This part was well done by most candidates. In some responses, candidates did not consider that equality held, writing only an inequality. In a handful of responses, candidates confused domain with range. In some responses, candidates did not solve $4x - 3 \geq 0$ correctly.

(ii) Almost all candidates found the correct expression for the inverse function, by the standard method of interchanging the x and y then changing the subject. In a few responses, candidates made manipulation errors, including $x = \sqrt{4y} - \sqrt{3} \Rightarrow x^2 = 4y - 3$.

(iii) In better responses, candidates found the points of intersection or at least stated $x = 1$ or $x = 3$. Candidates who solved $\sqrt{4x-3} = x$ were more likely to obtain the correct solution than those who attempted to solve $\sqrt{4x-3} = \frac{x^2+3}{4}$. Some obtained the solution by trial and error or by plotting points.

(iv) In better responses, candidates used an accurate scale on both axes and plotted the points they had found in parts (i) and (iii). In many weaker responses, candidates did not realise that their graphs should be mirror images in the line $y = x$ and, in this case, should be semi-parabolas. In some weaker responses, candidates did not restrict their graphs to the first quadrant.

(c) (i) In better responses, candidates first calculated the size of the sample space, which was 25. In many weaker responses, candidates ignored the possibility of a draw, even though it was in the question, and consequently stated that the probability of winning was 0.5.

(ii) A large majority of candidates gained full marks on this part by substituting their answer p from part (i) into the expression ${}^6C_3(p)^3(1-p)^3$.

(d) (i) This part was an unusual application of parameters. In better responses, candidates used the right angle given in the triangle to complete the proof successfully. The solution eluded many candidates who gave a circular proof by substituting $y = \frac{t^2}{k}$ into y. There were at least eight different approaches using the right angle, including gradients, Pythagoras' Theorem, similarity, distances, the equation of BC, areas and angles in a semicircle.

(ii) In a number of better responses, candidates deduced or stated correctly that $a = \frac{k}{4}$ and thus the focus was $\left(0, \frac{k}{4}\right)$. In weaker responses, candidates concluded that the focus was $\left(\frac{k}{4}, 0\right)$ or made other statements, suggesting a misunderstanding of the meaning of focal length.

Question 13

(a) Most candidates recognised that $\cos^{-1}\frac{2}{3}$ represented an angle. In many responses, candidates correctly used a double angle formula to find the answer. Some candidates used the *t*-results. In a significant number of weaker responses, candidates plugged the value into a calculator to produce an answer of $0.9938 \ldots$ For this part, candidates were asked to write the answer in the form $a\sqrt{b}$.

(b) (i) The two most common approaches that met with success were polynomial division or evaluating the limit as x approached infinity. While 2 was correctly identified by the majority of candidates, there did appear to be some confusion regarding what 'horizontal' meant. In some weaker responses, candidates came up with expressions such as '$x = 2$' or '$x^2 + 9 \neq 0$ and so $x \neq \pm 3$'. These values caused problems in the next part of the question when candidates tried to incorporate their answer into a graph.

(ii) In a significant number of weaker responses, candidates used calculus to locate stationary points, despite being told specifically 'Without the use of calculus'. In better responses, candidates incorporated their answer to part (i) into the graph. Some candidates, who were unable to find the horizontal asymptote in part (i), drew a correct graph in this part.

(c) (i) Candidates were asked to verify that the given expression satisfied the equation for simple harmonic motion. This required candidates to differentiate the expression twice and then apply a simple factorisation to complete the task. Most candidates handled this. However, establishing the centre of motion ($x = 5$) did provide a challenge for some. In many weaker responses, candidates left their answer in the form $\ddot{x} = -4(6\cos 2t + 8\sin 2t)$, which is not in the form required by the question, namely $\ddot{x} = -n^2(x - c)$.

(ii) Most candidates attempted to transform the given expression into a single trigonometric expression with a minority choosing to use t-results. The centre of motion not being at the origin caused problems for a significant number of candidates who reached conclusions such as displacement of the particle is zero at the centre of motion or that the acceleration is zero when displacement is zero. Finding the first time that the displacement is zero also presented problems for some candidates. Some candidates used degrees instead of radians. A minority of candidates chose to use $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4(x - 5)$. While this approach can lead to the correct answer, very few managed to complete the calculation.

(d) (i) In this part, candidates were required to use the product rule to differentiate an exponential function and then locate the stationary point. Most candidates gained full marks. A significant number of candidates wasted time proving that the lone stationary point was a maximum despite being given this information in the question.

(ii) While it was clear that most candidates could state Newton's method, only a few successfully applied it to find a correct approximation. In many weaker responses, candidates ignored the given value of 0.3.

Question 14

Many candidates scored well on this question. Candidates were required in different parts of the question to display clear geometric reasoning and strong interpretative skills, by making connections between different parts of the question and to solve a 3D problem. Many responses to parts (b) and (c) were marred by incorrect differentiation mainly through failing to distinguish between variables and constants.

(a) (i) Although many candidates successfully justified that $CTDS$ was a rectangle, some assumed it was sufficient to show that rectangles had either a pair of equal opposite or equal adjacent angles. In a number of responses, candidates unnecessarily showed that the opposite sides were parallel or tried to reason that CS was greater than CT. In the better responses, candidates used standard terminology when presenting their geometric justifications *f'(x)* or clearly explained the method they were attempting to use.

(ii) Although many candidates successfully completed this part, some did not use (i) to show that $SX = CX$ by using the fact that the diagonals in a rectangle are equal and bisect each other. In weaker responses, candidates often referred to 'properties of a rectangle' without specifying the properties they meant. A number of candidates assumed that ST was a tangent but rarely questioned this assumption when they were required to show that ST was a tangent in (iii). In some responses, candidates did not give supporting reasons for each step in their proof.

(iii) In many weaker responses, candidates did not provide full reasoning or see any contradictions in their reasoning with previous parts of the question, often leading to circular arguments. In some weaker responses, candidates gave inefficient reasoning about the angle in the alternate segment or equal tangents from an external point, and often did not explain why XC was a tangent first. In some weaker responses, candidates reasoned that ST was a tangent as it was perpendicular to the radius but did not provide clear reasoning. In some responses, candidates thought it sufficient to state that ST touched the semicircle once so it was a tangent to the semicircle. In better responses, candidates used the congruent triangles from (ii) to justify corresponding right angles and then correctly stated the relationship between a radius and the tangent at the point of contact.

(b) (i) In many responses, candidates recognised that maximum height occurred when $y' = 0$. They worked out a value for t and substituted the value for t into the parametric equation for y to find the maximum height. In a number of responses, candidates unnecessarily found the second derivative and justified that their t value was a maximum. In some weaker responses, candidates incorrectly differentiated the vertical displacement as $70\cos\theta - 9.8t$. A large number of candidates found the Cartesian equation first. These candidates often found the maximum height after inadvertently answering part (ii) with some not noticing this and reworking their solution in part (ii), sometimes with different reasoning. In some responses, candidates incorrectly differentiated the Cartesian equation by confusing variables and constants. Candidates who used the Cartesian approach were generally less successful, as were candidates who first found where the projectile hit the ground. In a smaller number of responses, candidates simply quoted a formula for the maximum height and did not show the result. Others merely applied physics formulae. A number of candidates derived the equations of motion despite having been given these. In some weaker responses, candidates took a rote approach to problem solving. A small number of attempts also included inappropriate approximations for g or t.

(ii) Most candidates who correctly answered part (i) completed this part easily. In some weaker responses, candidates simply quoted a formula for the maximum range, and did not satisfactorily establish their result.

(iii) Few candidates gained full marks on this part but many made some progress by attempting to solve the inequalities $125 < 250\sin 2\theta < 180$ and $250\sin^2\theta \geq 150$. In better responses, candidates found possible values of θ which satisfied one of the conditions for best viewing. In very few responses, candidates found the range of values that satisfied both inequalities. Generally speaking, those who solved the inequality $250\sin^2\theta \geq 150$ directly by taking the square root of both sides were much more successful than those who changed $\sin^2\theta$ to $\frac{1}{2}(1-\cos 2\theta)$. A number of responses were marred by solving equations rather than inequalities. In a large number of weaker responses, candidates unsuccessfully tried to solve this problem using a Cartesian approach and after pages of working still had not understood that the restrictions were on the maximum height and the corresponding horizontal distance. Candidates are reminded to look for links between question parts. Other candidates attempted to solve $125 < 70t\cos\theta < 180$ and $70t\sin\theta - 4.9t^2 \geq 150$, again ignoring the results of parts (i) and (ii). In many responses, candidates did not consider the domain of θ and/or 2θ and it was exceedingly rare for any candidate to consider graphs of $250\sin 2\theta$ or $250\sin^2\theta$. In weaker responses, candidates often reversed inequality signs or wrote an angle of 0.40 radians as 0.40π radians.

(c) (i) Many candidates found expressions for u and/or r by considering two appropriate triangles. In better responses, candidates included the use of the cosine rule in triangle BAG. In many weaker responses, candidates misused the cosine rule in triangle PAB. In most responses, candidates made some progress by correctly providing a relationship involving u or r. In weaker responses, candidates often interchanged the trigonometric ratios for BG and PG, and were unable to proceed further. In some weaker responses, candidates introduced extraneous variables for side names, for example, $AG = c$, without clarify the meaning of their variables either on a diagram or by a statement.

(ii) Candidates needed to apply the chain rule correctly to find the rate of change of r with respect to time. The use of a chain rule was often linked to incorrect variables such as $\frac{dr}{d\alpha}$. In many responses, candidates made some progress by finding $\frac{dr}{du}$ but a large number of candidates misinterpreted variables and constants and did not correctly differentiate. Few candidates completed this part by finding the correct value of $\frac{dr}{dt}$ as they did not find either the value of $\frac{du}{dt}$, required by the application of the chain rule, or the correct value of u with consistent units being applied. Many candidates found a correct expression for $\frac{dr}{dt}$ but omitted to substitute for u. In better responses, candidates used the expression given in the question rather than an expression obtained incorrectly in part (i).