Question 2

(a) $z = 2+3i \qquad w = 1+i$

$zw = (2+3i)(1+i)$

$= 2 + 2i + 3i + 3(i)^2$

$= 2 + 5i - 3$

$= \cancel{} -1 + 5i$

$\cancel{\dfrac{(1+i)}{(1+i)(1+i)}} = \dfrac{1}{(1+i)^2}$

$\dfrac{1}{w} = \dfrac{1}{1+i}$

$= \dfrac{(1-i)}{(1+i)(1-i)}$

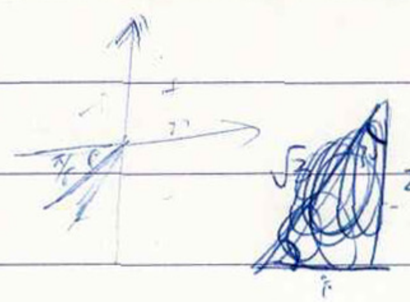
$= \dfrac{1-i}{1+1}$

$= \dfrac{1-i}{2}$

$= \frac{1}{2} - \frac{1}{2}i$

$\tan 60° = \sqrt{3}$

$60°, 180$



$\frac{9\pi}{3}$

$3\pi + \frac{\pi}{3}$

$2\pi = \frac{6\pi}{3} + \frac{4\pi}{3}$

$\frac{7\pi}{3} + \frac{\pi}{3}$

$\frac{10\pi}{3} =$

$3\pi + \frac{\pi}{3}$

(b) $z = 1 + \sqrt{3}i$

$|z| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\tan\theta = y/x = \sqrt{3}/1 = \sqrt{3}$

$\therefore \theta = \pi/3$

$\therefore z = 2(\cos \pi/3 + i\sin \pi/3)$

$(1+\sqrt{3}i)^{10} = z^{10} = 2^{10}\left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)$ By De Morv.

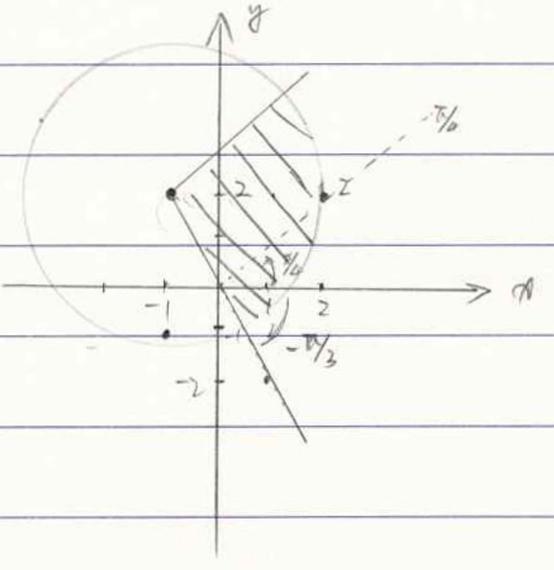
$= 2^{10}(-\sqrt{3} - \sqrt{3}i)$

$= -2^{10}\sqrt{3} - 2^{10}\sqrt{3}i$

c) $|z+1-2i| \le 3$ & $-\frac{\pi}{3} \le \arg z \le \pi/4$.

$|z-(-1+2i)| \le 3$

$z_0(-1,2)$



d) $z^4 = -1$ $\qquad \frac{2\pi}{4} = \frac{\pi}{2}$

$z = r(\cos\theta + i\sin\theta)$

$z^4 = (\cos\theta + i\sin\theta)^4$

$= \cos 4\theta + i\sin 4\theta$

$= -1$ $\qquad 0 \le \theta \le 360°$ ($0°$, 2π)

$\therefore \cos 4\theta = -1$ $\qquad 0° \le 4\theta \le 8\pi$.

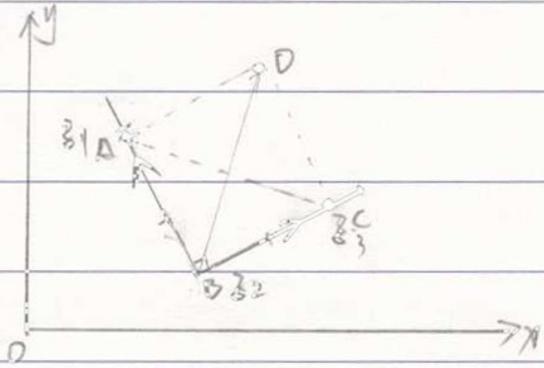
$4\theta = \cos^{-1}(-1) = \pi$

$4\theta = \pi, 3\pi, 5\pi, 7\pi$

$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$z = \cos\pi/4 + i\sin\pi/4$; $z_2 = \cos 3\pi/4 + i\sin 3\pi/4$; $z_3 = \cos 5\pi/4 + i\sin 5\pi/4$; $z_4 = \cos 7\pi/4 + i\sin 7\pi/4$

(e)



(i) In the diagram:

$z_1 - z_2 = \overrightarrow{BA}$

$z_3 - z_2 = \overrightarrow{BC}$

$(z_1 - z_2)^2 = BA^2$

$(z_3 - z_2)^2 = BC^2$

$\therefore$ $\because \triangle ABC$ is isosceles.

$\therefore$ $BA = CB$

$\therefore$ $(z_1 - z_2)^2 = -(z_3 - z_2)^2$

(ii) $\because$ ABCD is a square

$\therefore$ $BD = z_1 - z_2 + z_3 - z_2 = z_1 + z_3 - 2z_2$