a) $\frac{dV}{dt} = -k\sqrt{y}$ , $t=0$ , $y=y_0$

$t=T$ , $y=0$.

i) $\frac{dy}{dt} = \frac{dV}{dt} \cdot \frac{dy}{dV}$.

$V = Ay$

$\frac{dV}{dy} = A$

$\frac{dy}{dV} = \frac{1}{A}$

$\frac{dy}{dt} = \frac{-k\sqrt{y}}{A}$.

ii) $\frac{dt}{dy} = -\frac{A}{k\sqrt{y}}$

$\int_t^T dt = -A\int_{y_0}^{y} \frac{1}{k\sqrt{y}}\,dy$

$= \frac{A}{k}\int_y^{y_0} y^{-\frac{1}{2}}\,dy$

$= \frac{2A}{k}\left[y^{\frac{1}{2}}\right]_y^{y_0}$

$T - t = \frac{2A}{k}\left(y_0^{\frac{1}{2}} - y^{\frac{1}{2}}\right)$

$\sqrt{y_0} - \left(\frac{k(T-t)}{2A}\right) = \sqrt{y}$.

$\cancel{y_0 - \frac{k^2(T-t)^2}{4A^2} = y}$.

$y_0\left(\frac{K(T-t)}{2A}\right) = y$

(iii) $t = 10 \quad , \quad y = \frac{y_0}{2}$

~~t = T~~

$t = T \qquad y = 0$

$$\frac{\cancel{y_0}}{2} = \cancel{y_0}\left(1 - \frac{10}{T}\right)$$

$$1 - \frac{1}{2} = \frac{10}{T}$$

$$\frac{1}{2} = \frac{10}{T}$$

$\therefore T = 5\text{ s}$

$\therefore$ 5 seconds

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b) $z_n = \cos(\alpha + n\beta) + i\sin(\alpha + n\beta)$

i)

$z_0 = \cos\alpha + i\sin\alpha$

$z_1 = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$ ∴ also $z_n$.

!

the ext < of a polygon with equal sides and [illegible] they are all equal.

$\overrightarrow{OP_0}$ ~~$z_0$~~

$\overrightarrow{P_0P_1} = z_1$

$\overrightarrow{P_1P_2} = z_2$.

$\overrightarrow{P_3P_2} = z_3$.

since $z_1 = z_2 = z_3$ (equi distance)

& ~~$z_3 \angle z$~~

$\theta_0 = z_3 - z_2 > z_1 - z_0$ ~~$z_0$~~

$\theta_2 = z_2 - z_1 - z_0$

$\theta_1 = z_1 - z_0$

$= z_0$

(b) (iii) In $\Delta OP_0P_1$.

$P_1P_0 = OP_0$ (given)

$\angle P_1OP_0 = \angle OP_1P_0$ (base $\angle$s isos $\Delta$)

$= \alpha$.

$\theta_0 = 2\alpha$ (ext. $\angle$ of $\Delta$)

$\alpha = \frac{\theta_0}{2}$

In $\Delta P_0P_1P_2$.

$P_0P_1 = P_1P_2$ (given)

$= OP_0$

$\angle P_0P_2P_1 = \angle P_1P_0P_2$ (base $\angle$s isos $\Delta$)

$= \beta$.

$\theta_1 = 2\beta$

$\beta = \frac{\theta_1}{2}$.

since $\theta_0 = \theta_1$ (from i)

$\alpha = \beta$

i.e. $\angle P_0OP_1 = \angle P_0P_2P_1$.

b) ii)

~~[illegible]~~

$OP_0P_1P_2$ cyclic quad

$\Rightarrow \angle P_0OP_1 = \angle P_0P_2P_1$

$\rightarrow \angle$ subtends from the same chord.

iii) In $\triangle P_1P_2P_3$,

Similarly,

~~$\angle P_2P_1P_3 =$~~

$\angle P_2P_3P_1 = \angle P_2P_0P_1$

(~~because~~ base $\angle$s of congruent isos $\triangle$s & $\theta_1 = \theta_2$)

& $P_0P_1P_2P_3$ is cyclic quad.

let $\angle P_3P_2O = \delta$

$\delta = \pi - \theta_2 - \angle P_0P_2P_1$

$= \pi - \theta_2 - \alpha$

$\angle P_3P_1O = \pi - \theta_0 - \angle P_2P_0P_1$

$= \pi - \theta_0 - \alpha$ (since $\angle P_0P_2P_1 = \angle P_2P_0P_1$ from ii)

$= \pi - \theta_2 - \alpha$ (since $\theta_0 = \theta_2$ from i)

$= \angle P_3P_2O$.

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(iii) $OP_0P_1P_2P_3$ are ~~conc~~

concyclic →

∠ subtend from the

same segment.

(iv) $z_0 = \cos\alpha + i\sin\alpha = \text{cis}$

$z_1 = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$z_2 = \cos(\alpha+2\beta) + i\sin(\alpha+2\beta)$

$z_3 = \cos(\alpha+3\beta) + i\sin(\alpha+3\beta)$

$z_4 = \cos(\alpha+4\beta) + i\sin(\alpha+4\beta)$

$z_n = \text{cis}(\alpha + n\beta)$

$z_0 + z_1 + z_2 + z_3 + z_4 = \text{cis}(\alpha +$

$= \text{cis}\,\alpha + \text{cis}(\alpha+\beta) + \text{cis}(\alpha+2\beta) + \text{cis}(\alpha+3\beta) + \text{cis}(\alpha+4\beta) = 0$

$= ~~5\text{cis}\,\alpha~~\ 5\cos\alpha +$

~~equate real coefficients parts~~

$5\cos\alpha = 0.$

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Q7 b) i)

$\alpha\cancel{\beta_0} =$

$\dot{\theta}$